Low Ply Drawings of Trees of Bounded Degree

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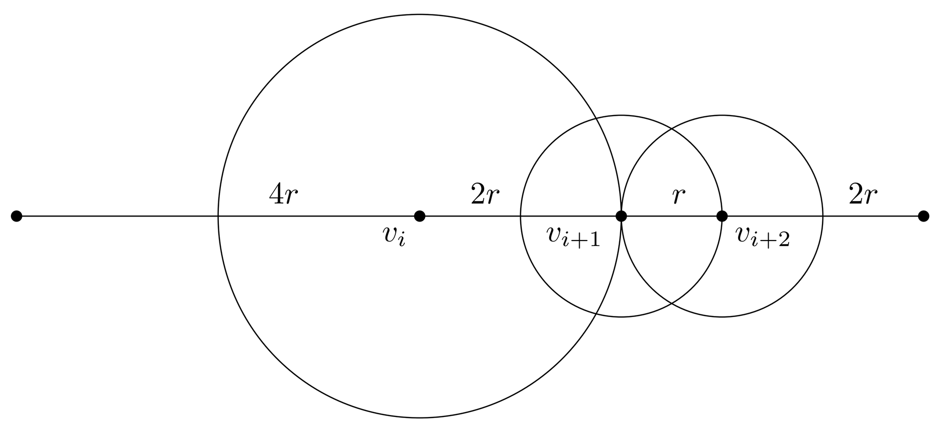
Overview

Ply number is a recently developed graph drawing metric inspired by studying road networks, which are known to have low ply [3]. For each vertex, we draw a open disk with radius α times the length of the longest incident edge. The ply number is the largest number of disks that intersect at a single point.

Usually, α is chosen in the range (0, 0.5], so that a graph with a single edge between two vertices has ply number 1.

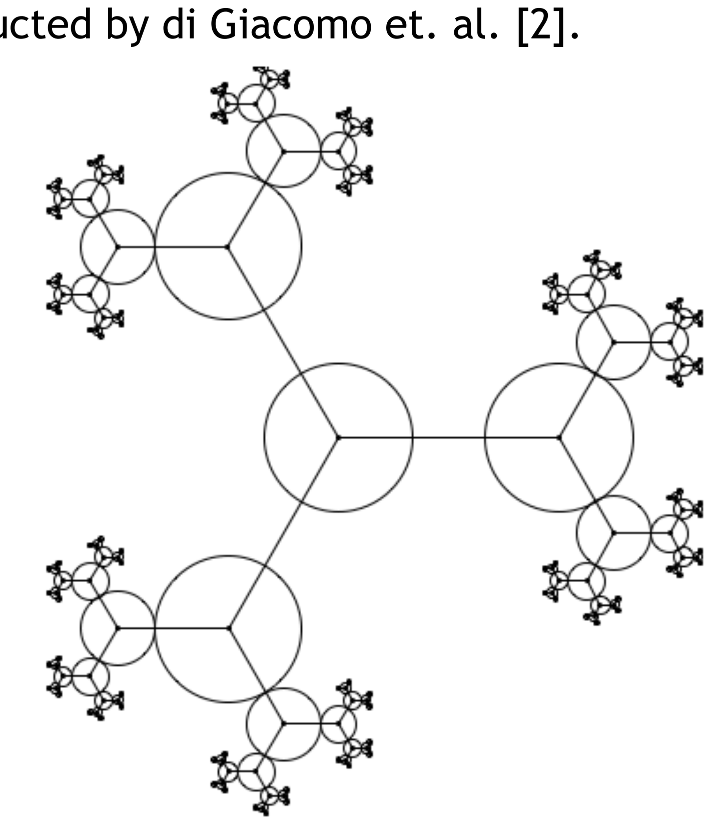


A graph drawing and its ply disks. α = 0.5, and the ply number is 2.

We first resolve an open problem proposed by di Giacomo et. al. [2], and show that for a sufficiently small α, any bounded-degree tree can be drawn with ply number 1. We then improve a result from Angelini et. al [1]. They showed that a degree-six tree can be drawn with α = 0.5 and ply number O(log n) in polynomial area; we extend this result to trees of any bounded degree.

1-ply Drawings

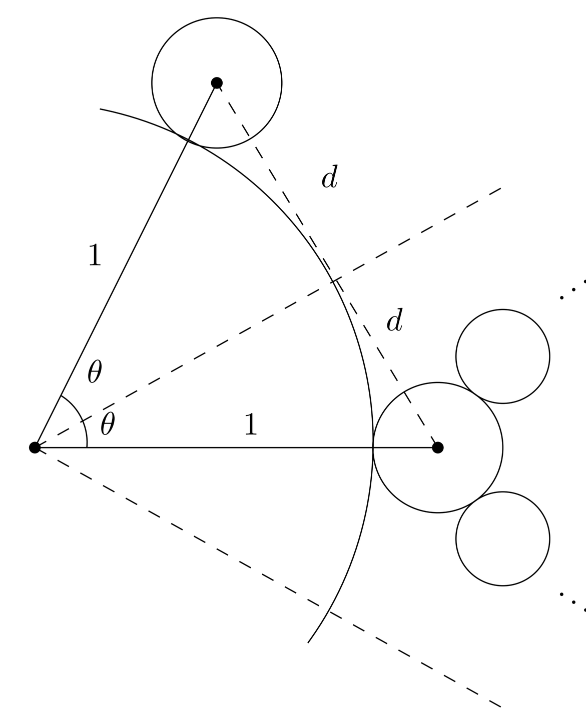
The following 1-ply drawing of a binary tree with α = 1/3 was constructed by di Giacomo et. al. [2].



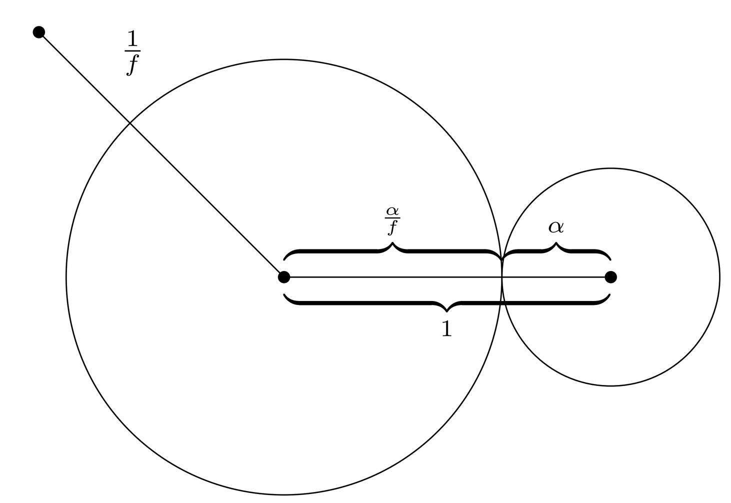
We mimic this drawing style to draw trees of any bounded degree. For a tree with maximum degree Δ, we divide the area around each parent vertex radially into Δ equal wedges. Then we draw one subtree inside each wedge. The distance from each node to its children is chosen to be a constant fraction f of its distance from its own parent.

This drawing would produce a fractal in the limit that contains every tree with maximum degree Δ. We then identify three conditions on α and f such that no two ply disks overlap.

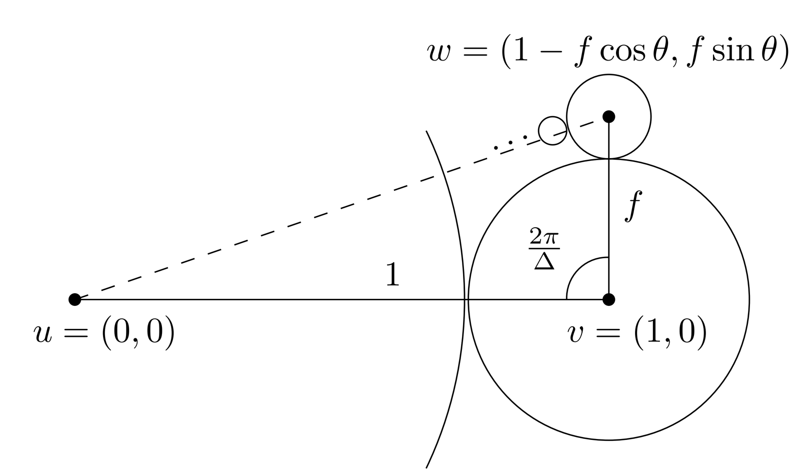
1-ply Drawings (cont.)



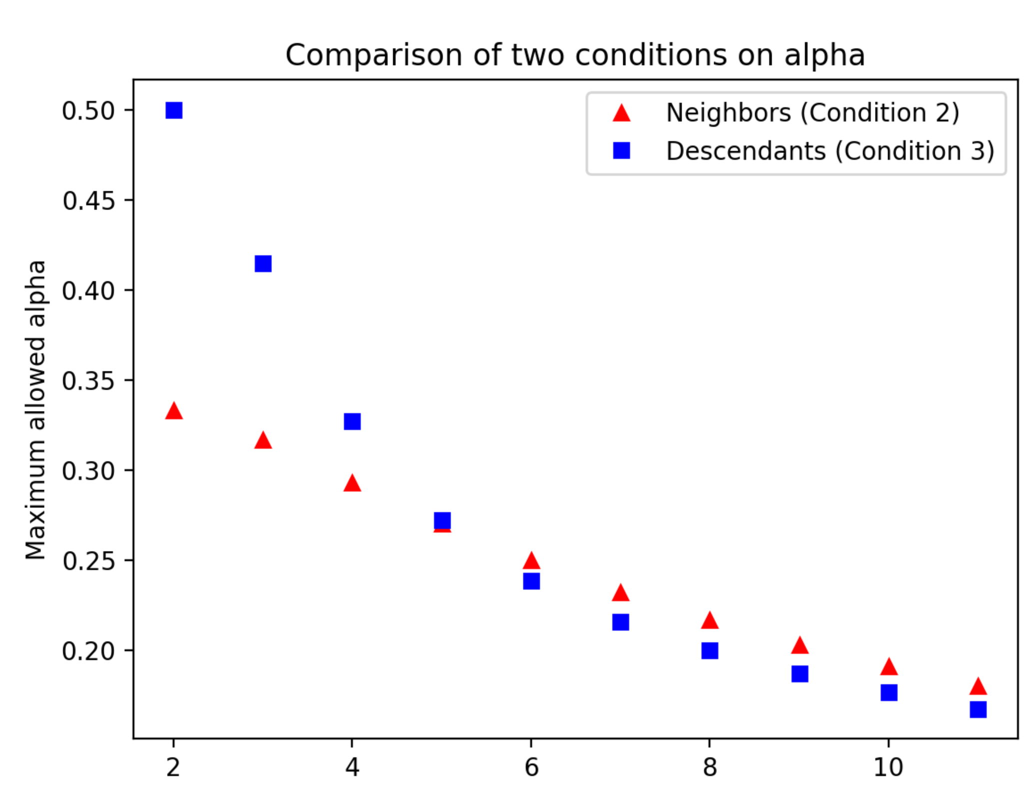
Condition 1: Ply disks must be contained within their wedge.



Condition 2: Ply disks for adjacent vertices cannot overlap.



Condition 3: Ply disks must not overlap an ancestor.

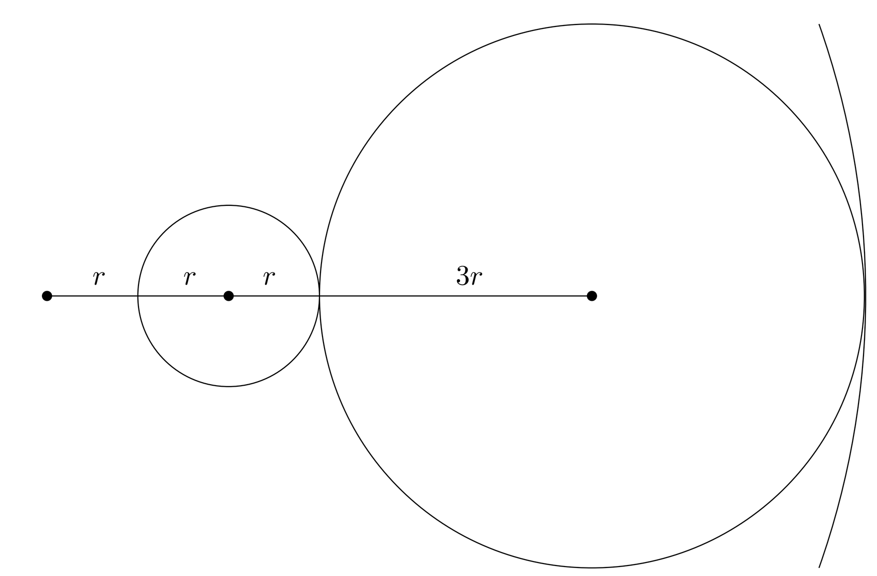


Condition 1 gives an upper bound for f, which we set to be as large as possible. Then we solve conditions 2 and 3 to determine the maximum value allowed for α.

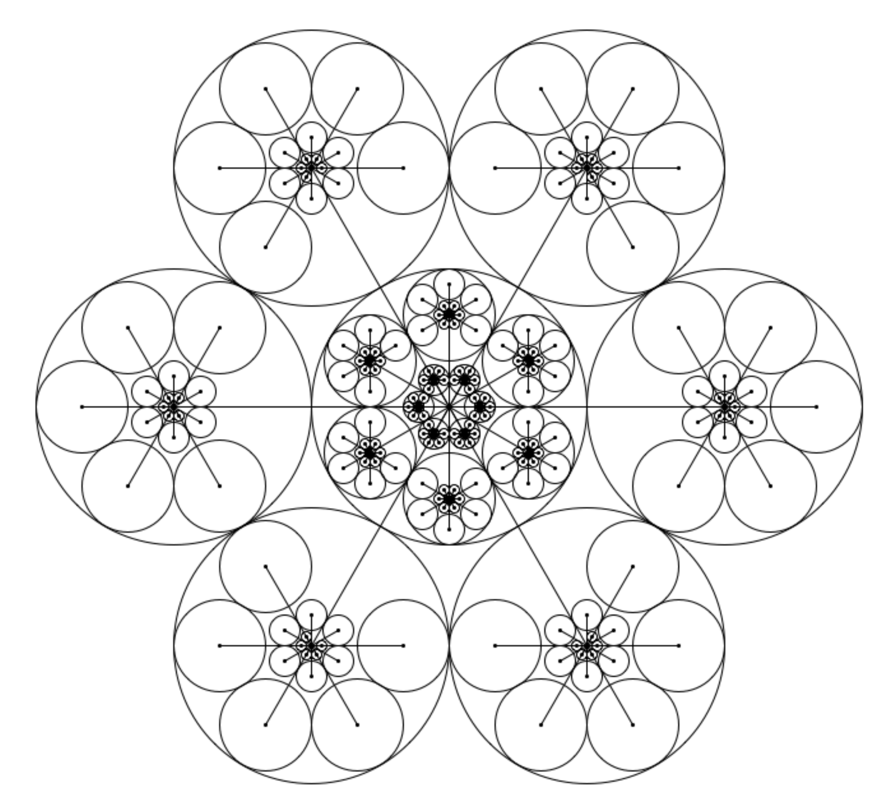
Layering Drawings for Log Ply

We now turn to drawings of trees in which α = 0.5. It is known that 10-ary trees cannot be drawn with constant ply number in this case [1], so we allow ourselves O(log n) ply number.

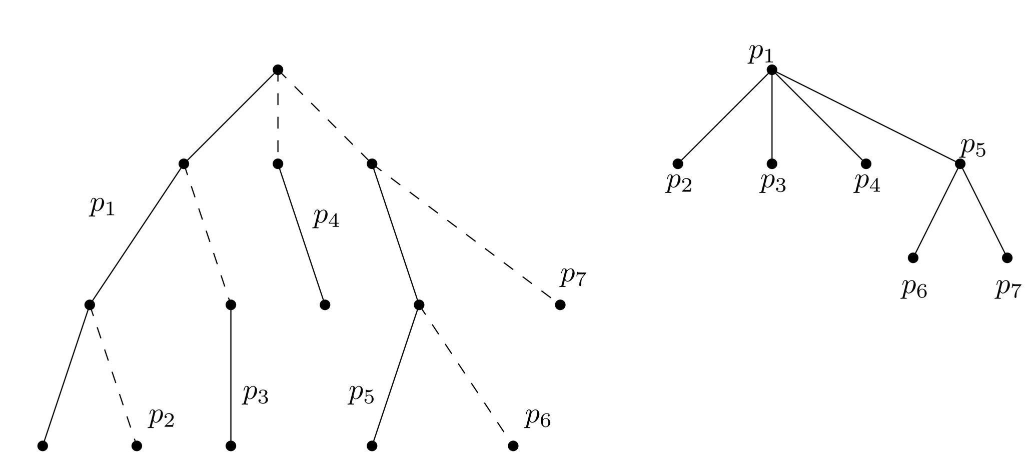
We first show that this can be achieved for balanced trees of any bounded degree in polynomial area. Then we use the heavy path decomposition as in [1] to achieve this result for trees that are not necessarily balanced.



Each layer is drawn at a distance of three times the previous layer.

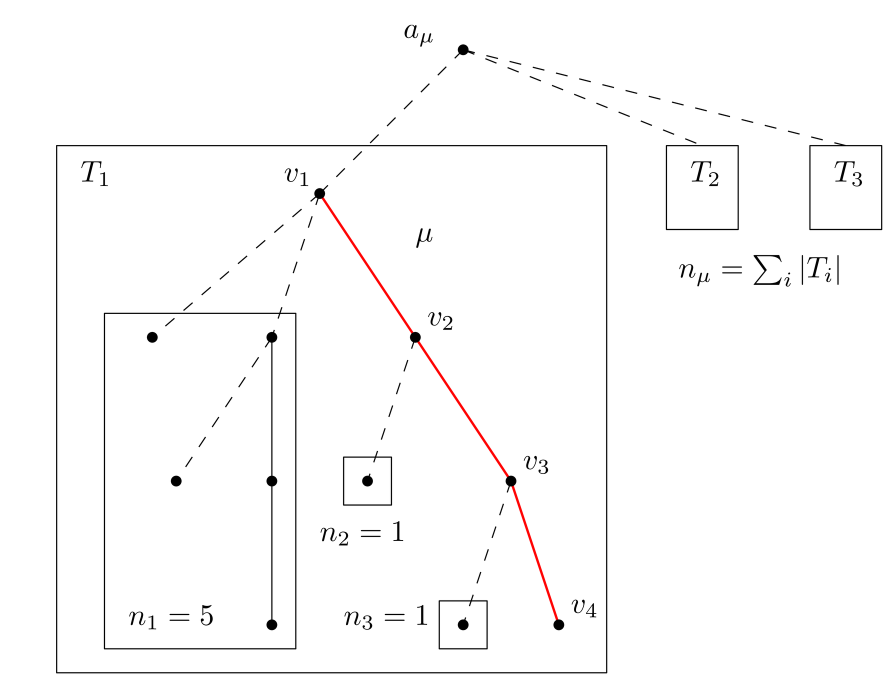


A tree with degree eighteen, with each level drawn in three layers.



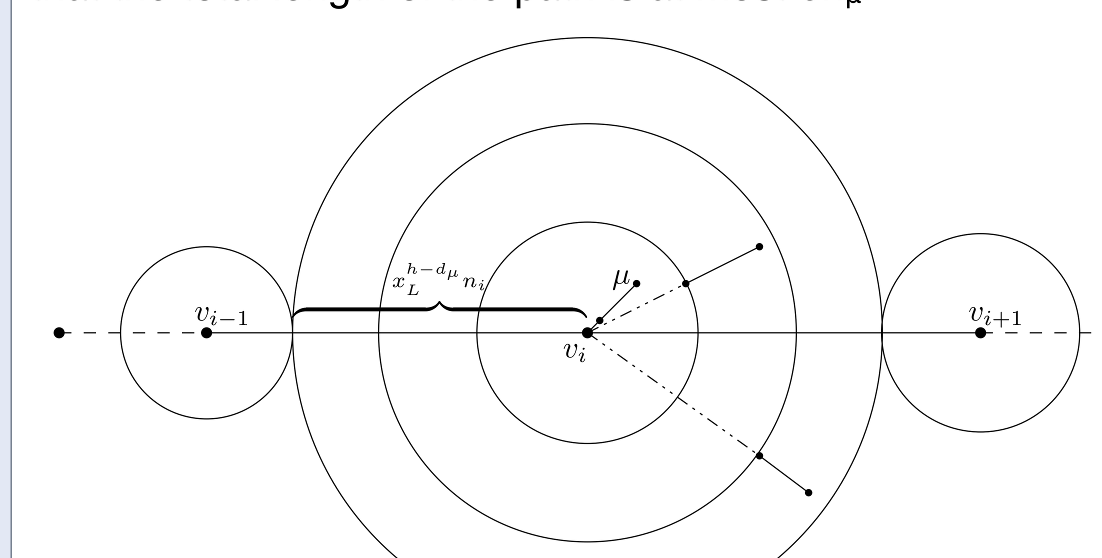
A tree and its heavy path decomposition.

Layering Drawings for Log Ply (cont.)



A path μ, and a labeled division of its subtrees.

We now draw each path in a straight line as follows. First, set $l(a_\mu, v_1) = n_1$, $l(v_i, v_{i+1}) = n_i + n_{i+1}$. Then visit the edges in order of decreasing length, and increase the length of any edge that is less than half that of its neighbors. It is shown in [1] that the ply number of this path drawing is at most 2, and that the total length of the path is at most $6n_\mu$.



To draw a tree in our heavy path decomposition, we put the drawings for each of its subtrees in a different layer. We put the root of each child path at least three times the distance of any vertex in the previous layer, so no two child paths have overlapping ply disks. Call the outer radius of the largest layer x_L. Each layer will be scaled up by a factor of x_L from its children, so that none of the paths anchored at adjacent vertices overlap.

References

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