Optimal Compaction of Orthogonal Grid Drawings for Graphs of Arbitrary Vertex Degrees

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Objective

Develop a method to perform the NP-complete compaction step of the TSM approach on planar graphs of arbitrary vertex degree. Graphs with vertex degree greater than four are represented following the Kandinsky approach[2], and the compaction method is an extension of Klau and Mutzel's ILP formulation[1].

The method must yield orthogonal grid drawings which are edge-length optimal.

Conclusions

The profusion of data created and collected that define this digital age often translate into big, dense graphs. Thus, being able to obtain an orthogonal, edge-length optimal, grid drawing for any planar graph is of great benefit to any application aiming for robustness, efficiency and flexibility.

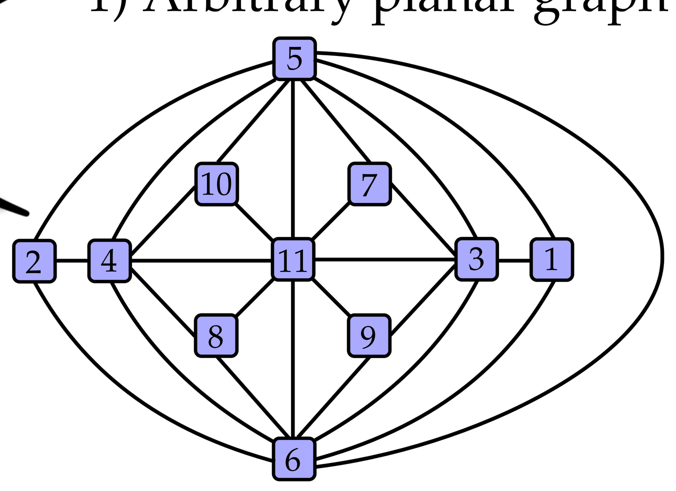
Computationally, the increased vertex degree does not compromise the overall performance of the procedure, since the - polynomial - preprocess stage becomes more efficient.

Main Contribution:

Capability of *optimally* compacting an orthogonal grid drawing for *any* planar graph.

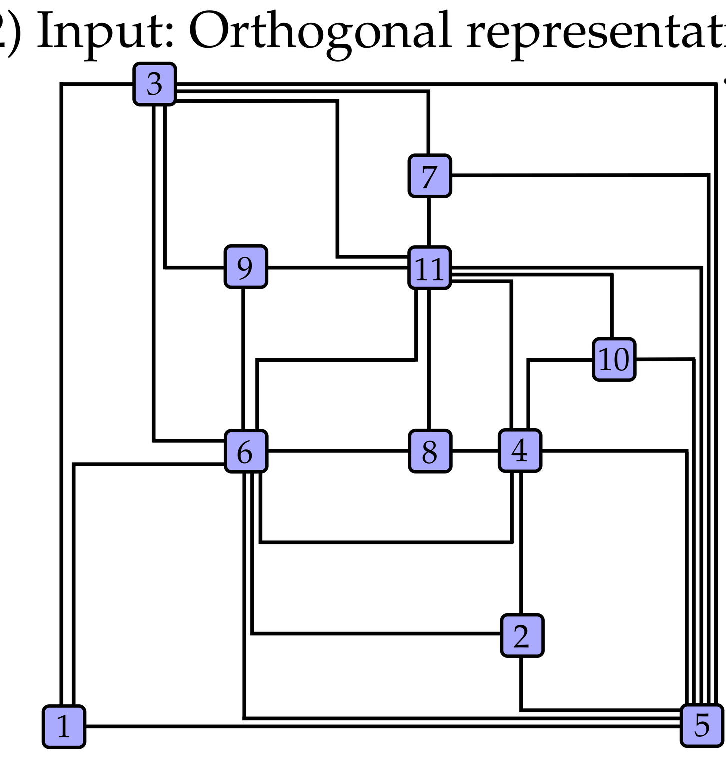
Methodology

1) Arbitrary planar graph

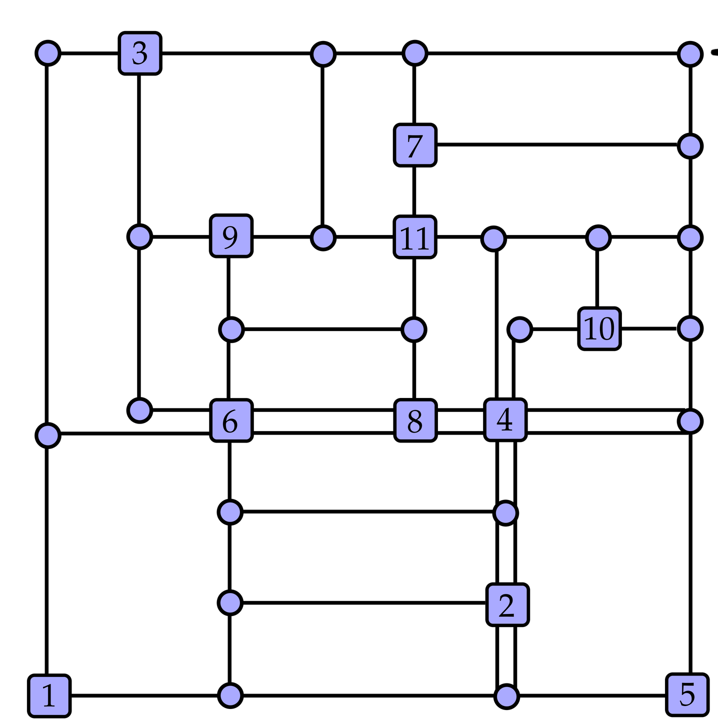


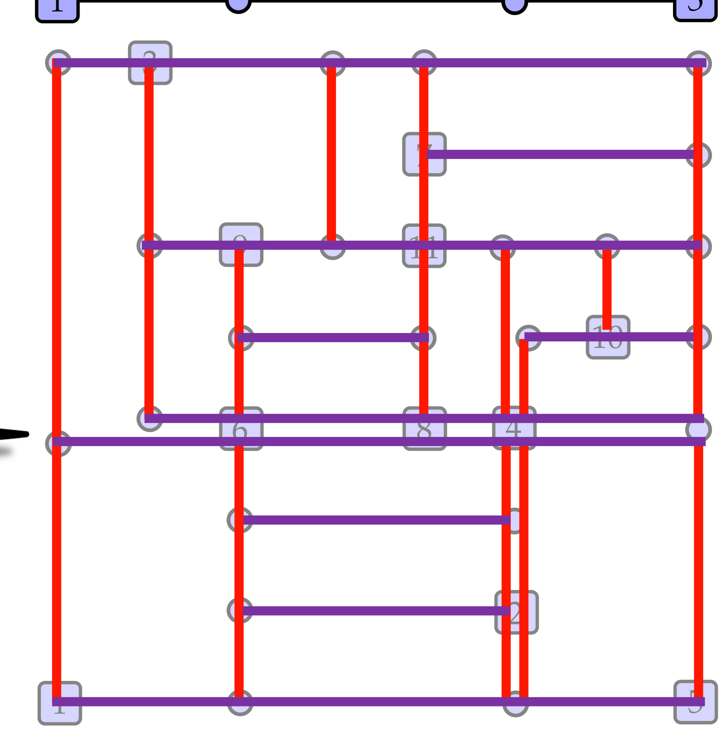
Goldner-Harary graph

2) Input: Orthogonal representation



3) Dummy vertices insertion, union of parallel edges and segments.

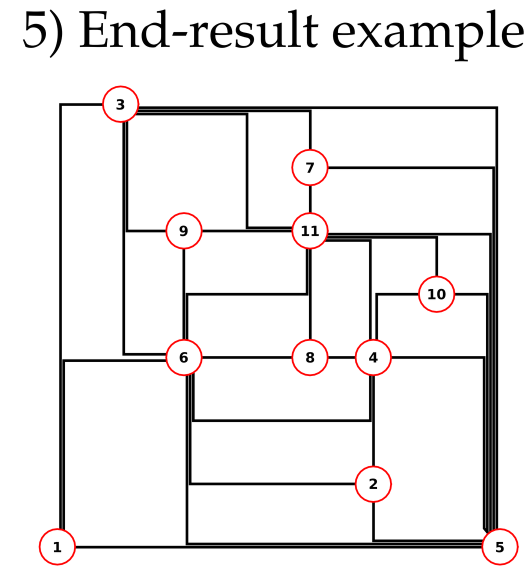




4) Integer Linear Programming formulation:

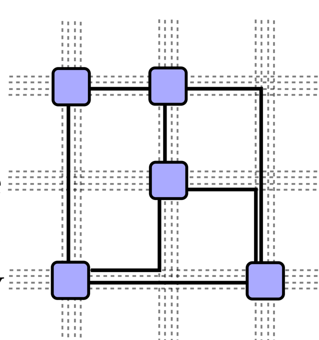
New constraint: every pair of horizontal (resp. vertical) twin segments must occupy the same vertical (resp. horizontal) grid position.

5) End-result example



Foundation #1: Kandinsky Model[2]

The Kandinsky Model is an efficient and aesthetically-pleasing way of representing graphs with vertex degree greater than 4 on the grid. In it, the grid lines are coarse, able to hold multiple edges side by side; in consequence, 0^{o} angles between to successive edges incident on a vertex are allowed, as long as the so-called bend-or-end property is not violated.



Additional Ideas:

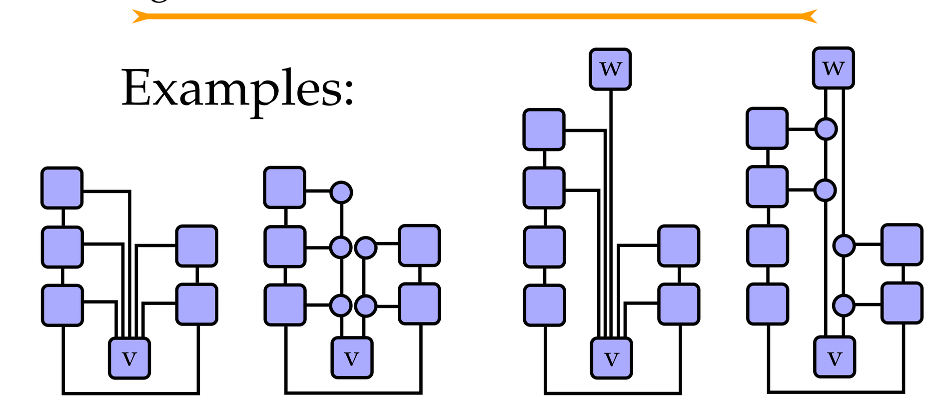
Aggregating Segments

All edges that are incident to the same side of the vertex and that bend to the same side may be joined in a single, "**aggregating**", segment. Straight edges are also included in these segments.

Twin Segments

When edges incident on the same vertex side bend to opposite directions, in order to guarantee correctness and optimality, two "**twin**" segments are required. If there are straight edges, they compose both segments.

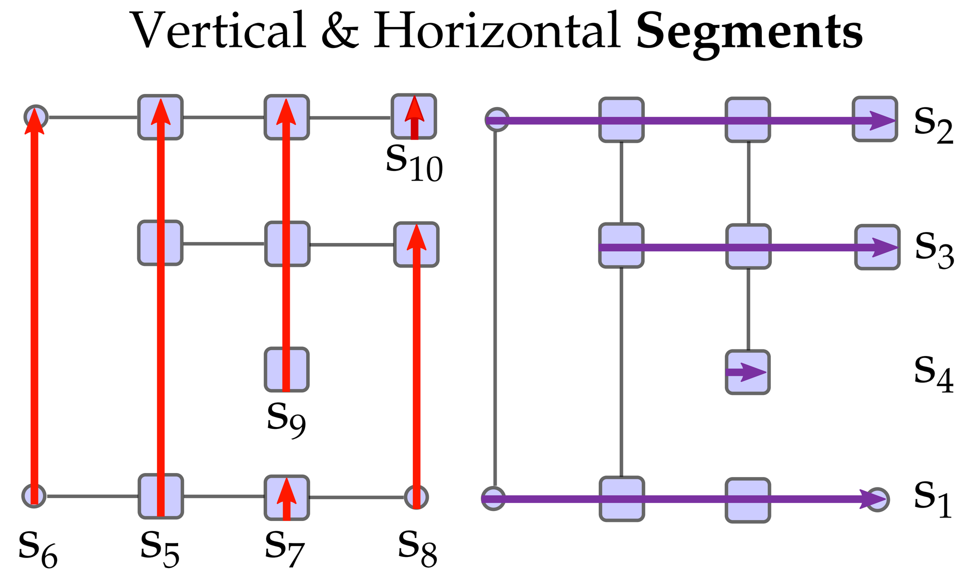
Examples:



Foundation #2: Klau and Mutzel's Optimal Compaction Method for 4-Planar Graphs[1]

Input: Orthogonal representation

Vertical & Horizontal **Segments**



Shape Description

ILP Formulation:

Minimize the total edge-length of the graph, *subject to*:
(i) every **non-adjacent** segment must be **separated**;
(ii) separations defined by the shape description must be enforced.

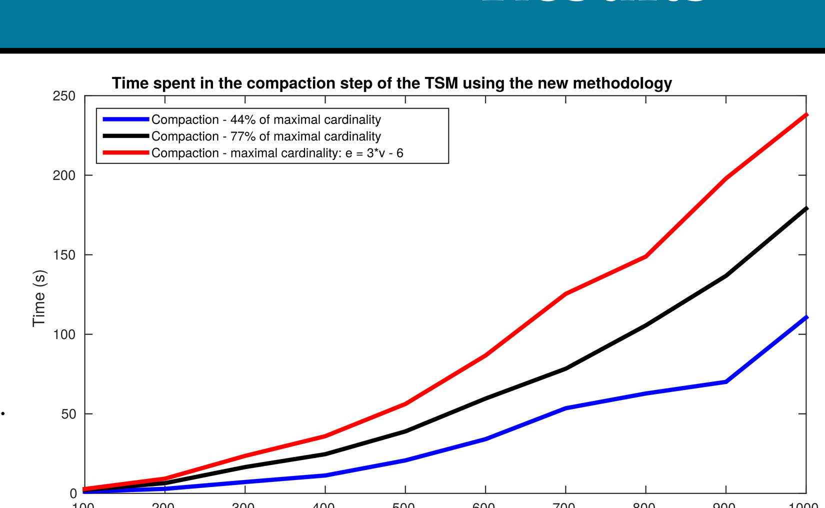
ILP Solution:

Optimal horizontal (resp. vertical) location of vertical (resp. horizontal) segments, i.e., an *edge-length optimal drawing*.

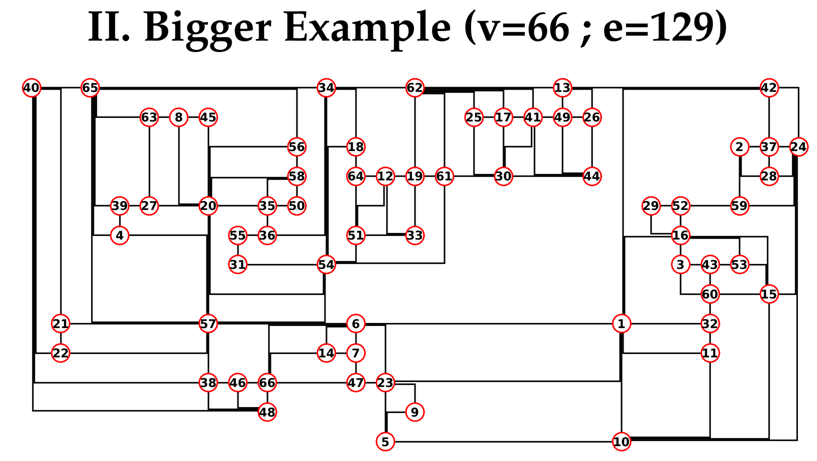
Results

I. Computational Behavior

Randomly generated examples of prescribed densities show that, for bigger cardinalities, the behavior of the compaction (NP-complete) presents behavior closer to polynomial than to exponential.



II. Bigger Example (v=66 ; e=129)



References

[1] Klau, G.W., Mutzel, P.: Optimal compaction of orthogonal grid drawings. Integer Programming and Combinatorial Optimization (1999)

[2] Fößmeier, U., Kaufmann, M.: Drawing high degree graphs with low bend numbers. Proceedings of the Symposium on Graph Drawing (GD'95) (1996)

Acknowledgment

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