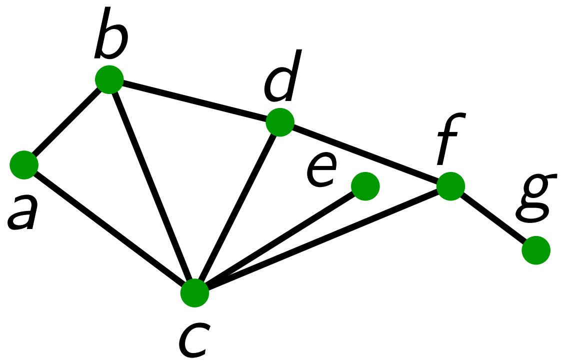
# Mixed Linear Layouts of Planar Graphs

**Sergey Pupyrev**

# Linear Layouts



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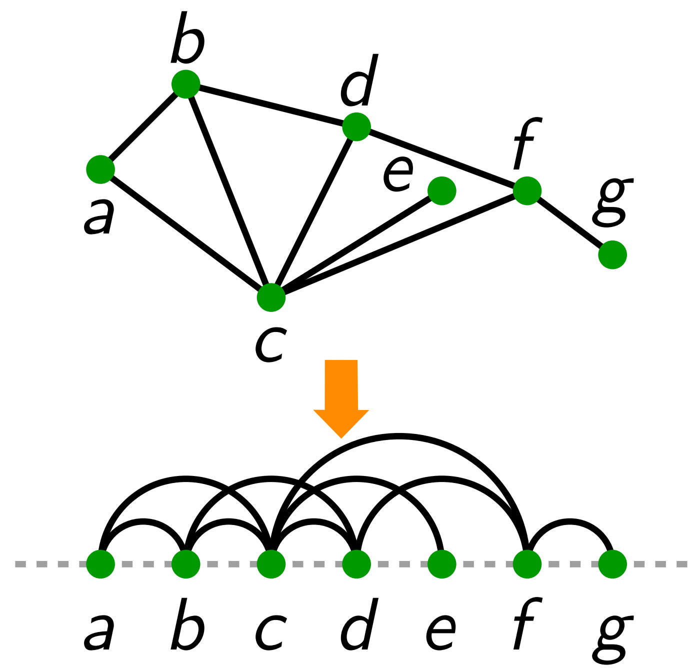

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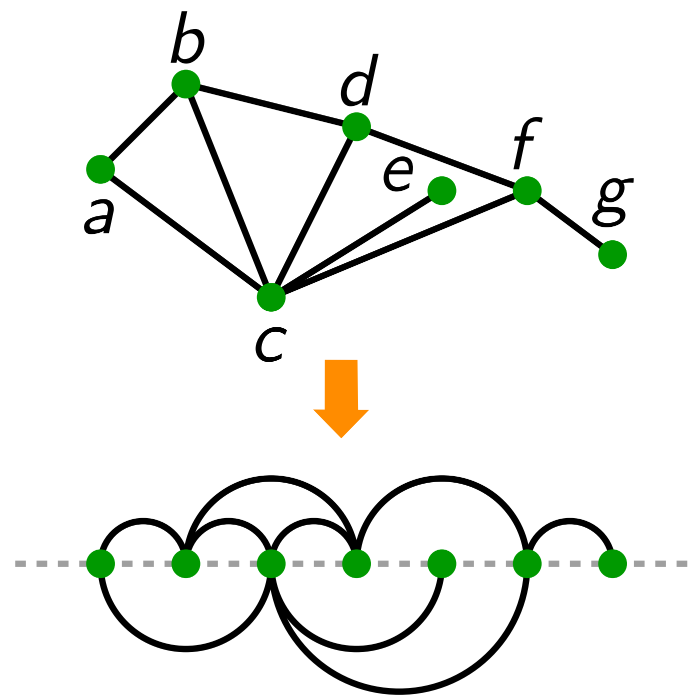

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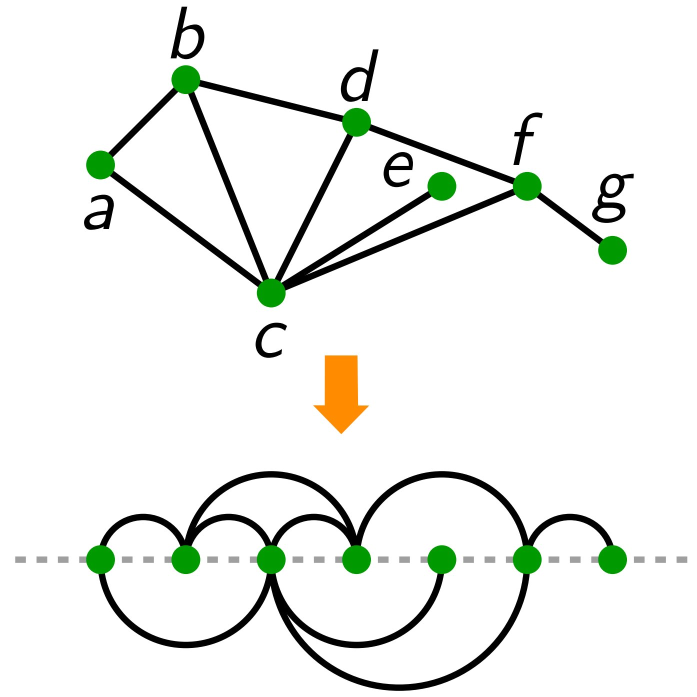


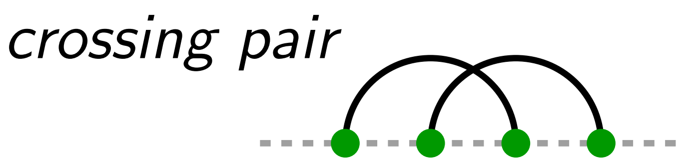

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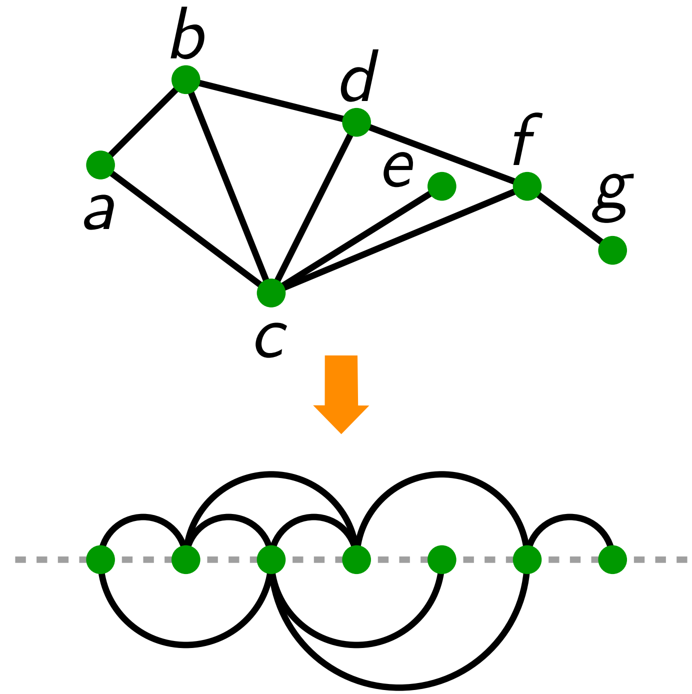
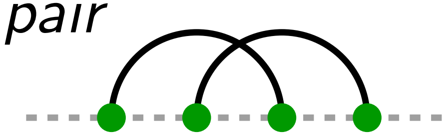
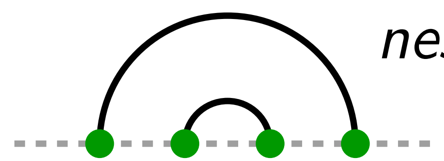



crossing pair



nested pair



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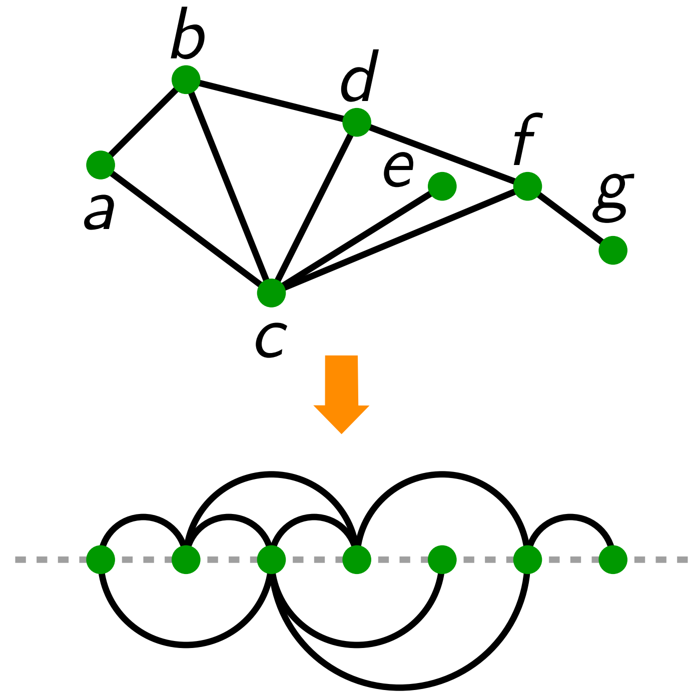
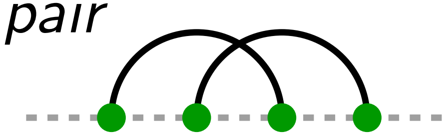



*crossing pair*



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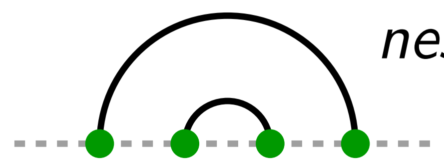

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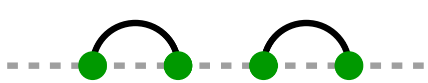

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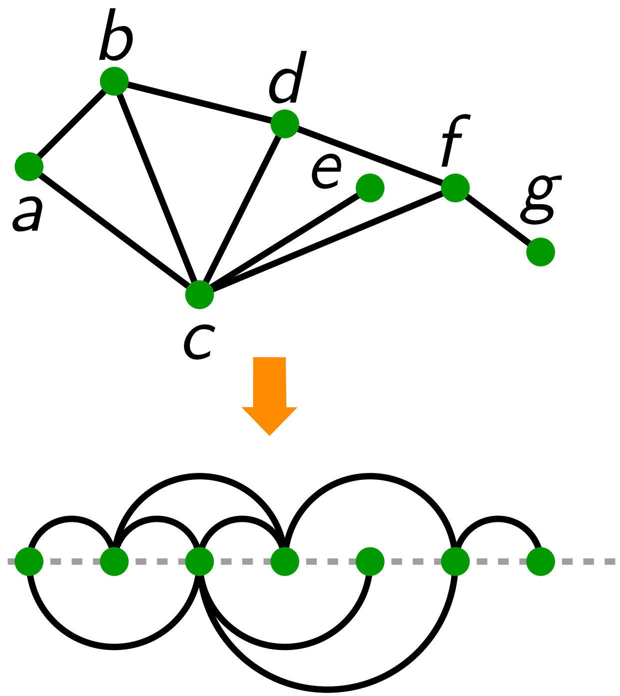


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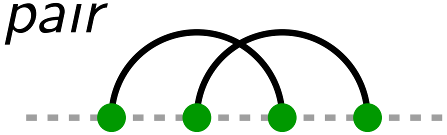

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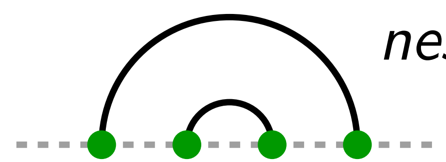

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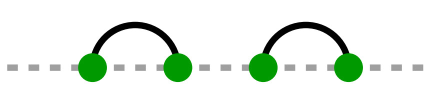
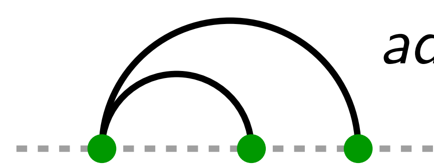


*adjacent pair*



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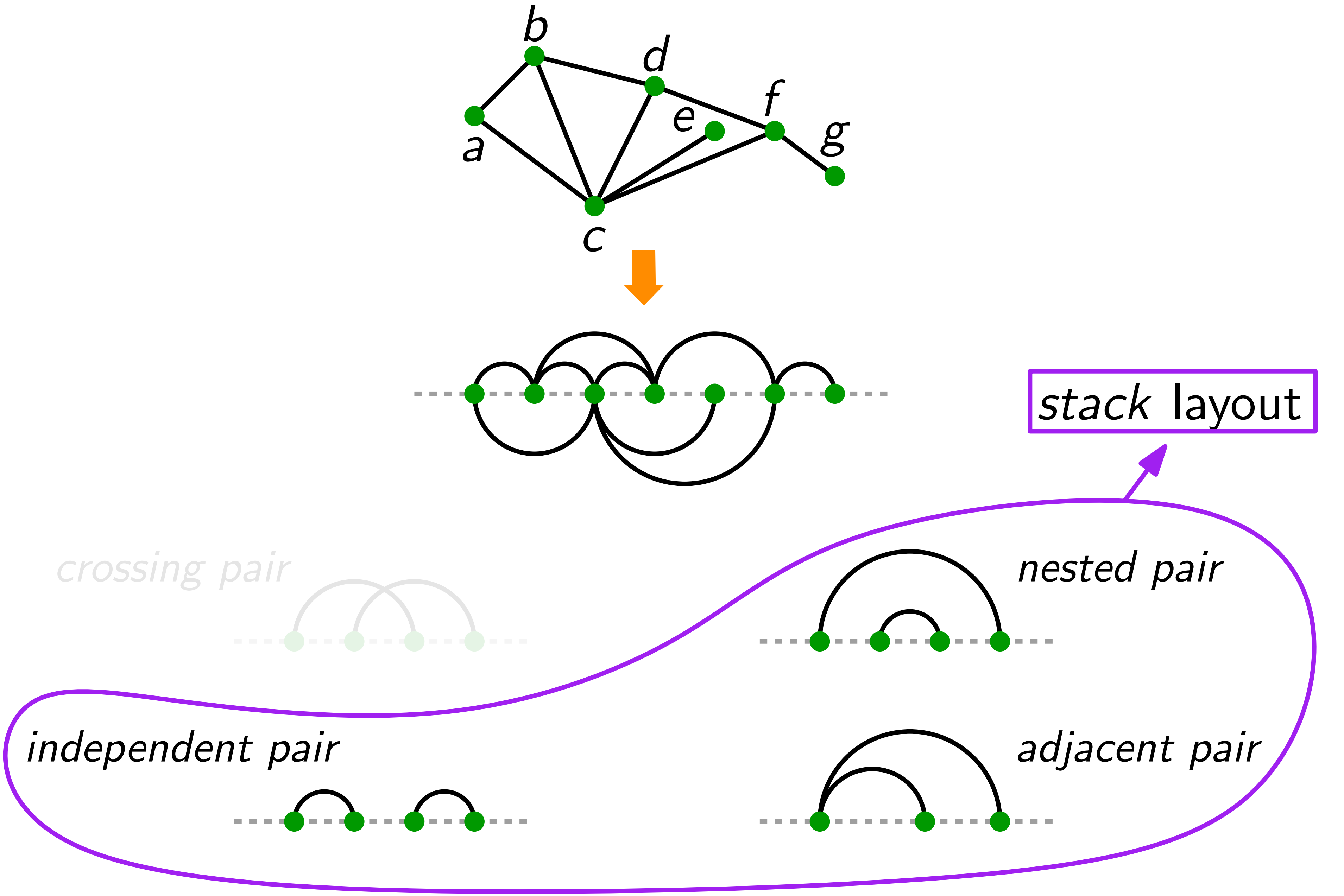

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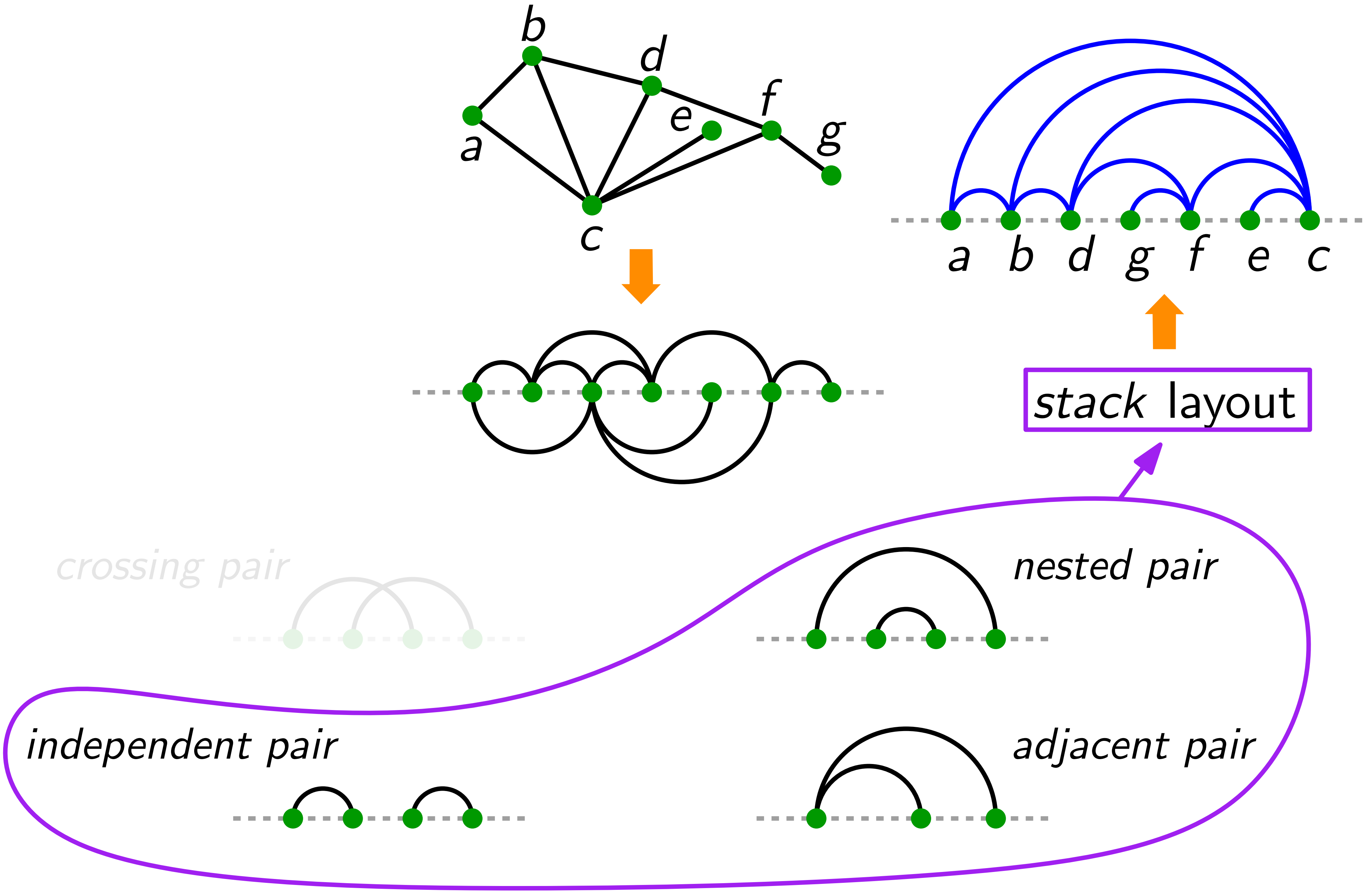

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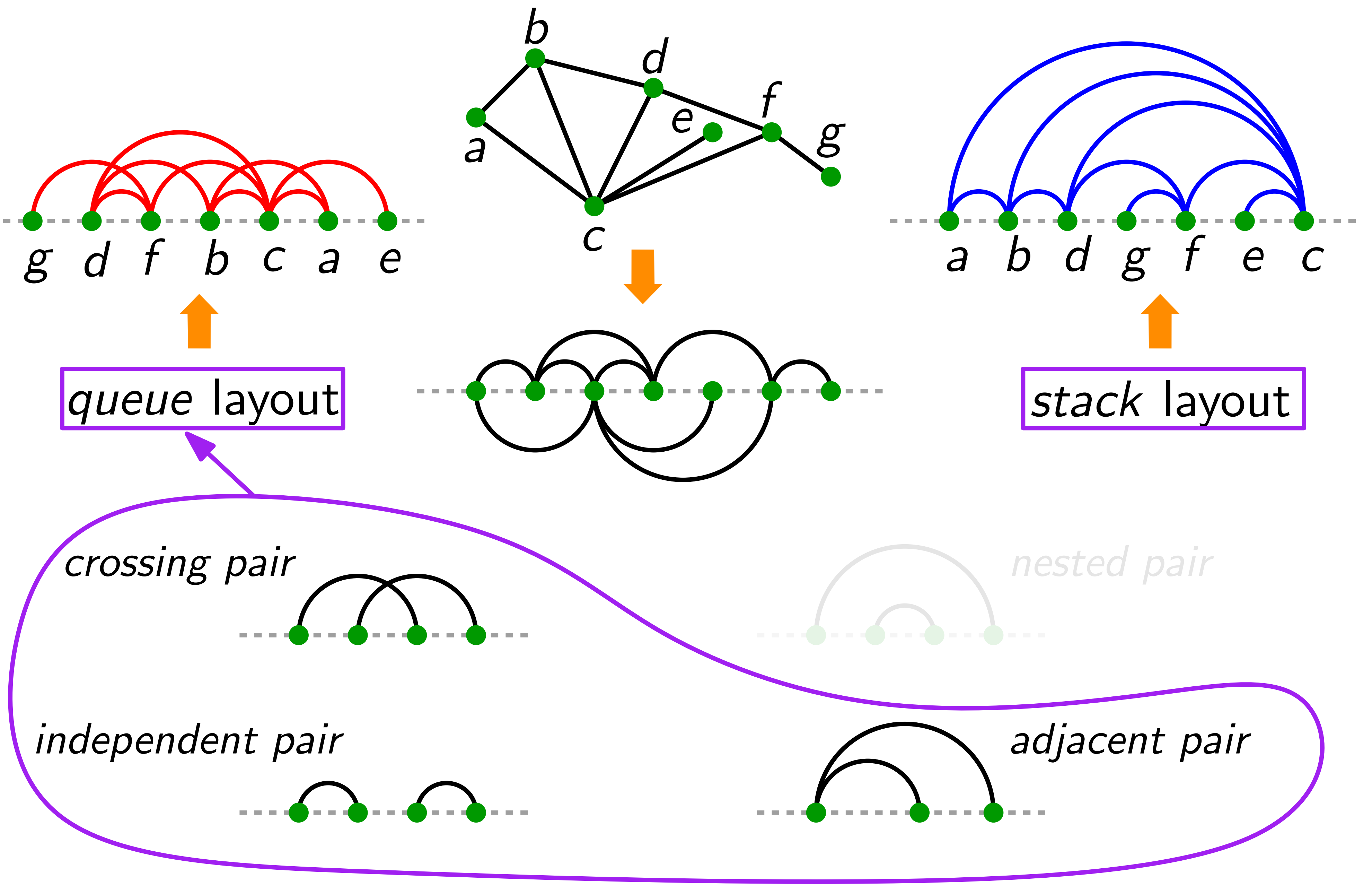

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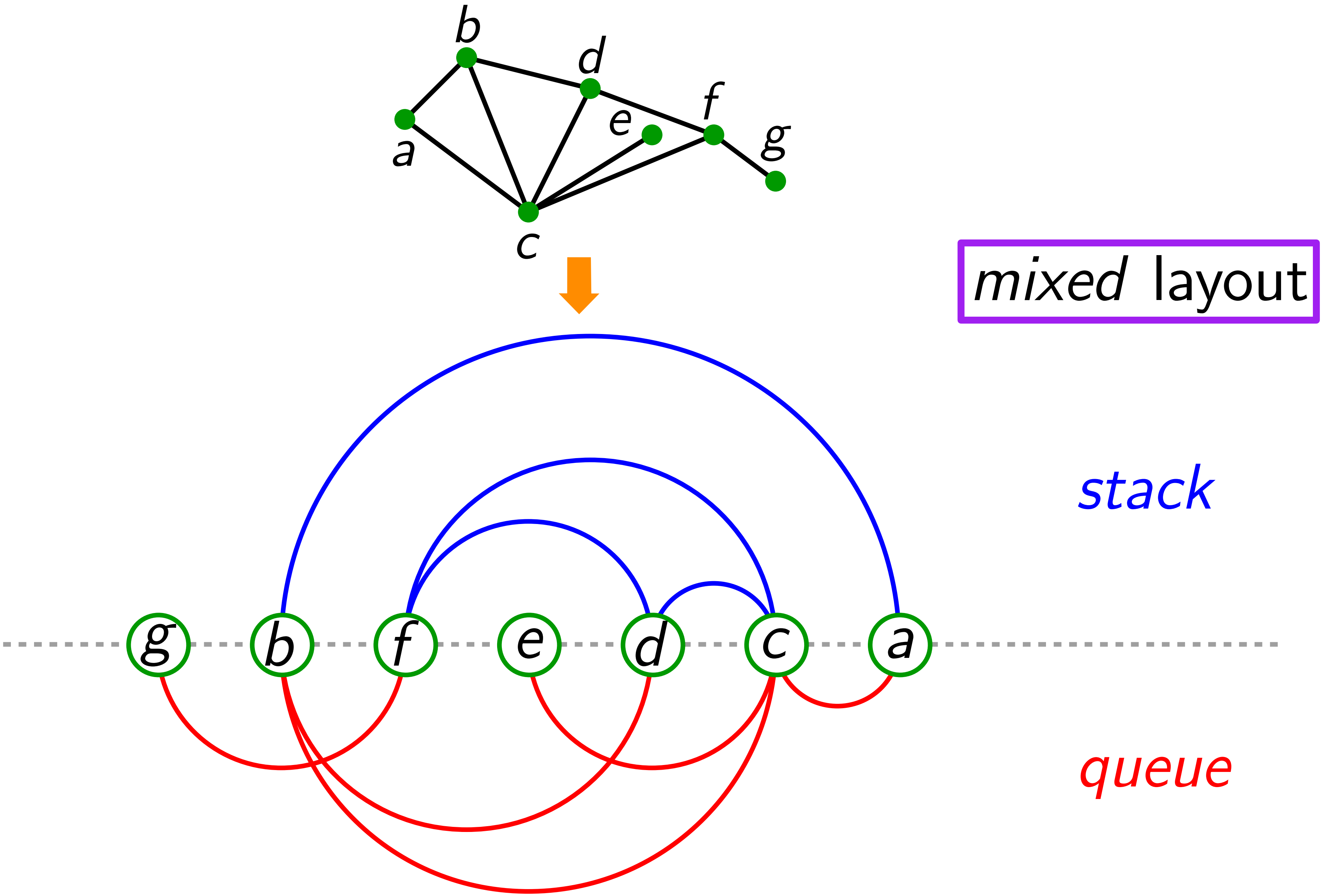

# Prior Work

| *stack* number | graph class | *queue* number |
|---|---|---|
| **1** | tree | **1** |
| | outerplanar | |
| | series-parallel | |
| | planar-3-tree | |
| | planar | |

# Prior Work

| *stack* number | graph class | *queue* number |
|---|---|---|
| **1** | tree | **1** |
| **1** | outerplanar | **2** |
| | series-parallel | |
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Every planar graph admits a mixed stack+queue layout

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There exists a planar graph that **does not admit** a mixed 1-stack 1-queue layout

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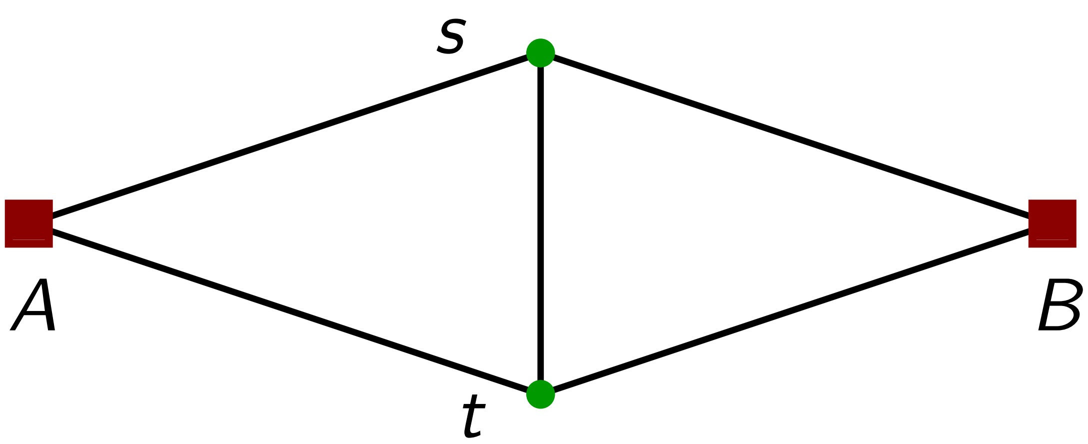
**Theorem 1**

There exists a planar graph that **does not admit** a mixed 1-stack 1-queue layout

**Theorem 2**

Every planar graph **admits** a mixed 1-stack 1-queue **subdivision** with one division vertex per edge

# Counterexample



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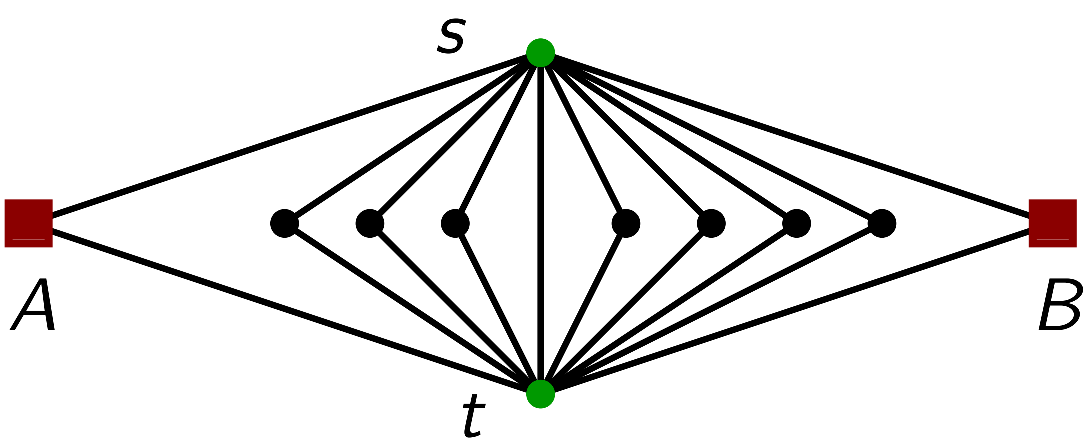

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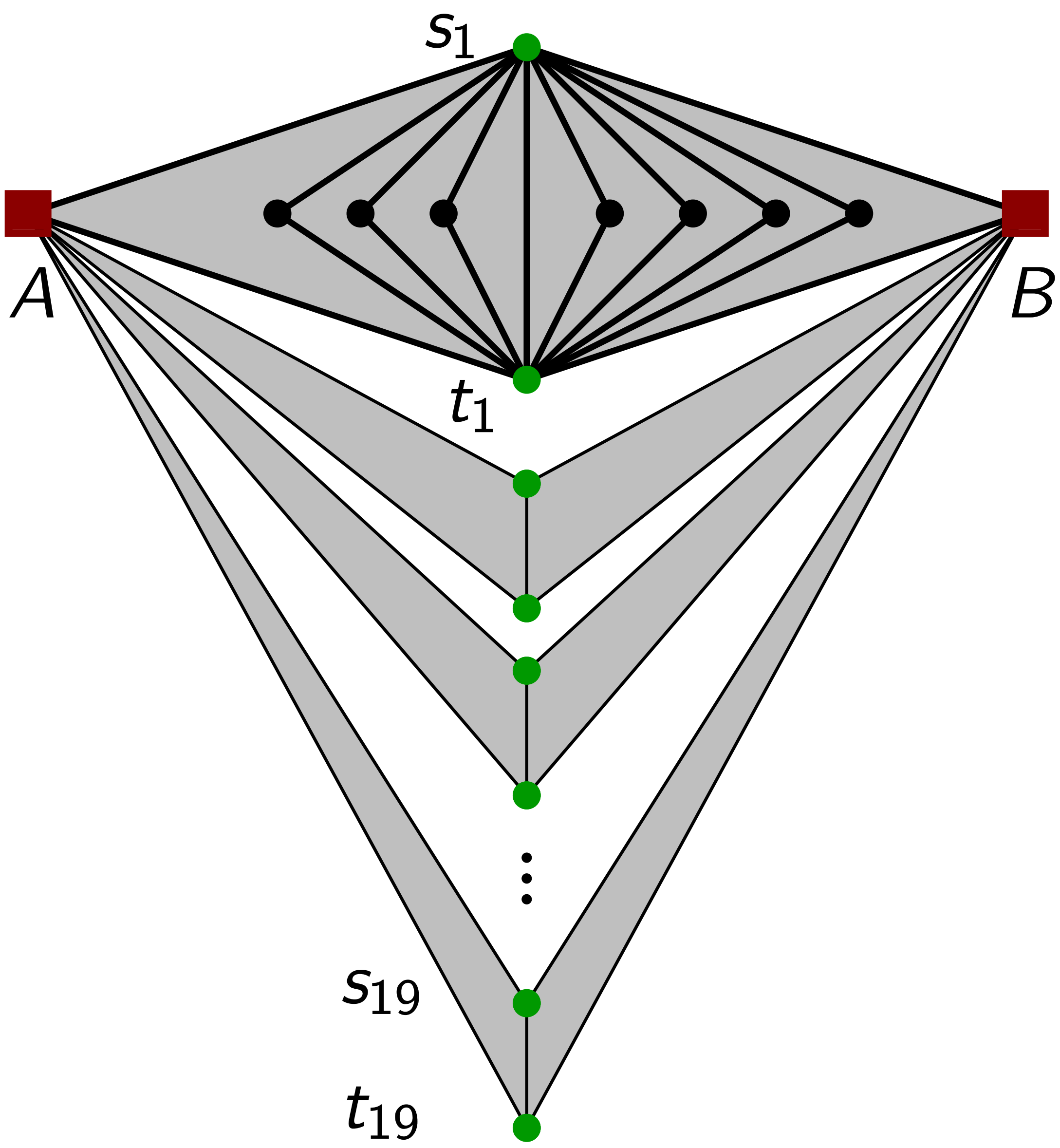

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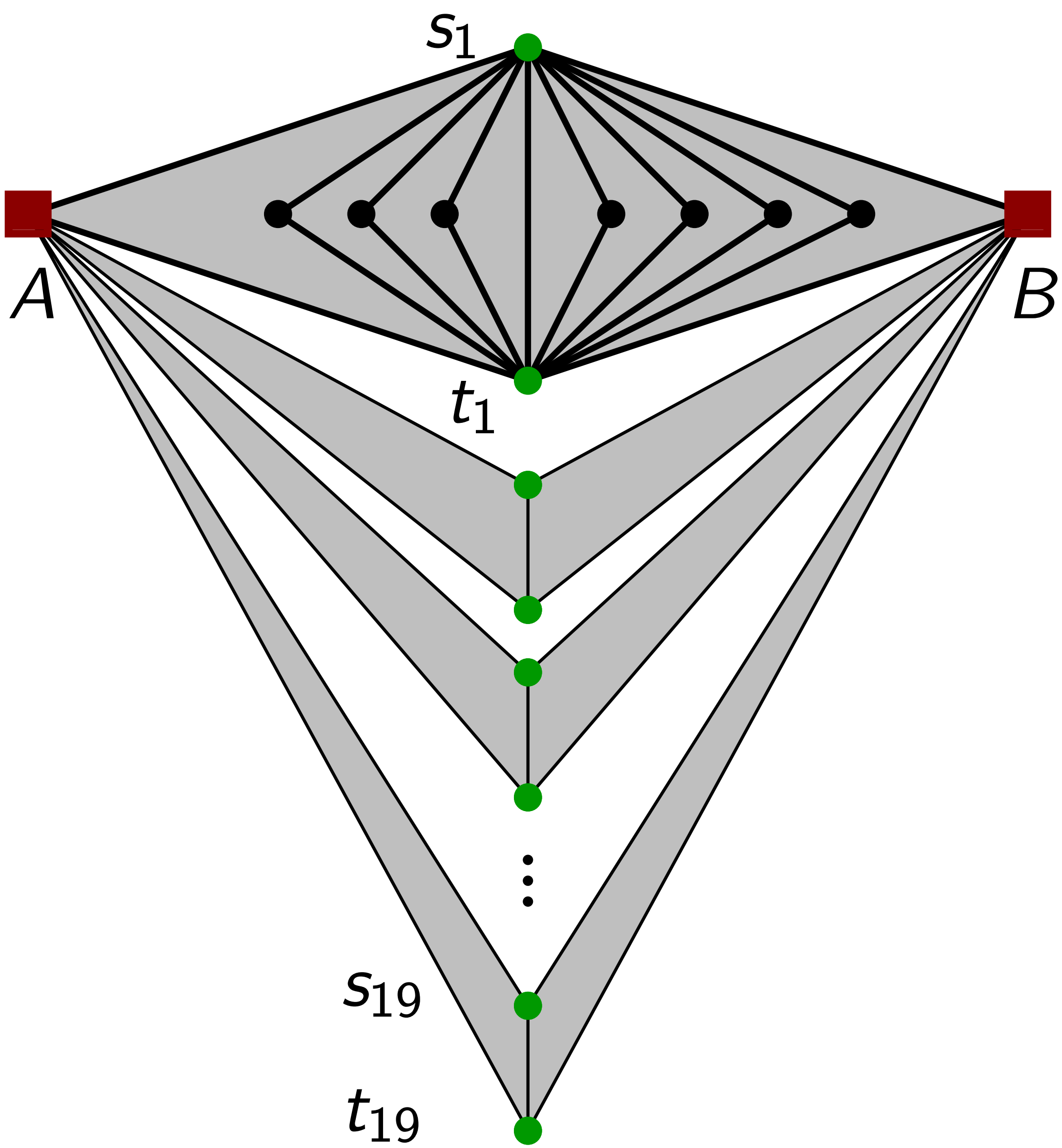


possible solution:

$A < B$ $s_i < t_i$

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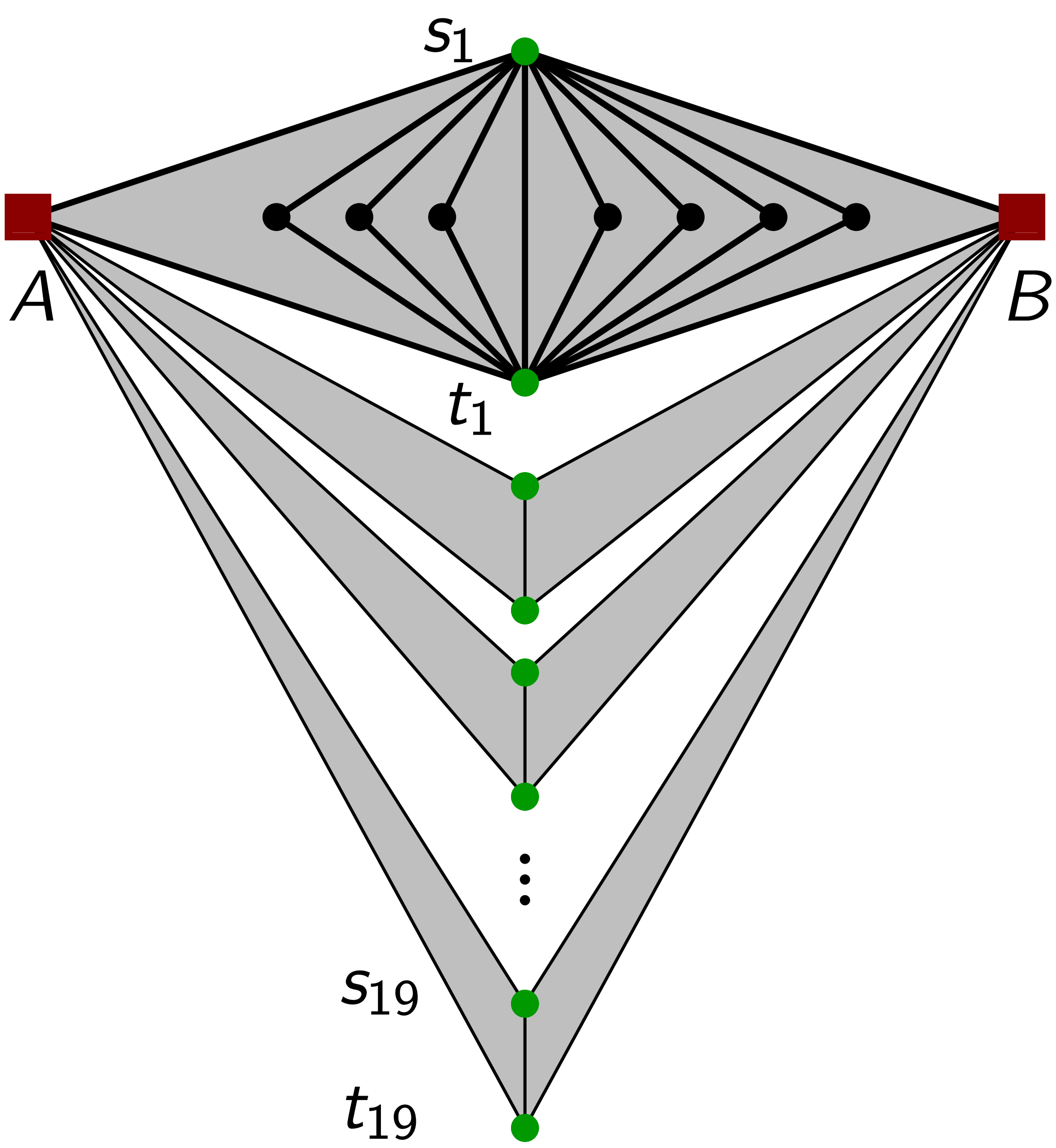


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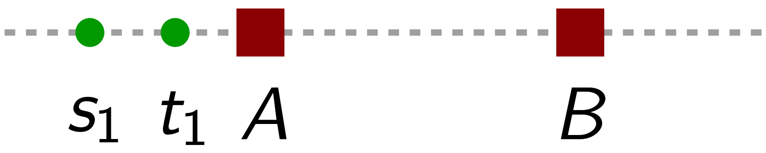

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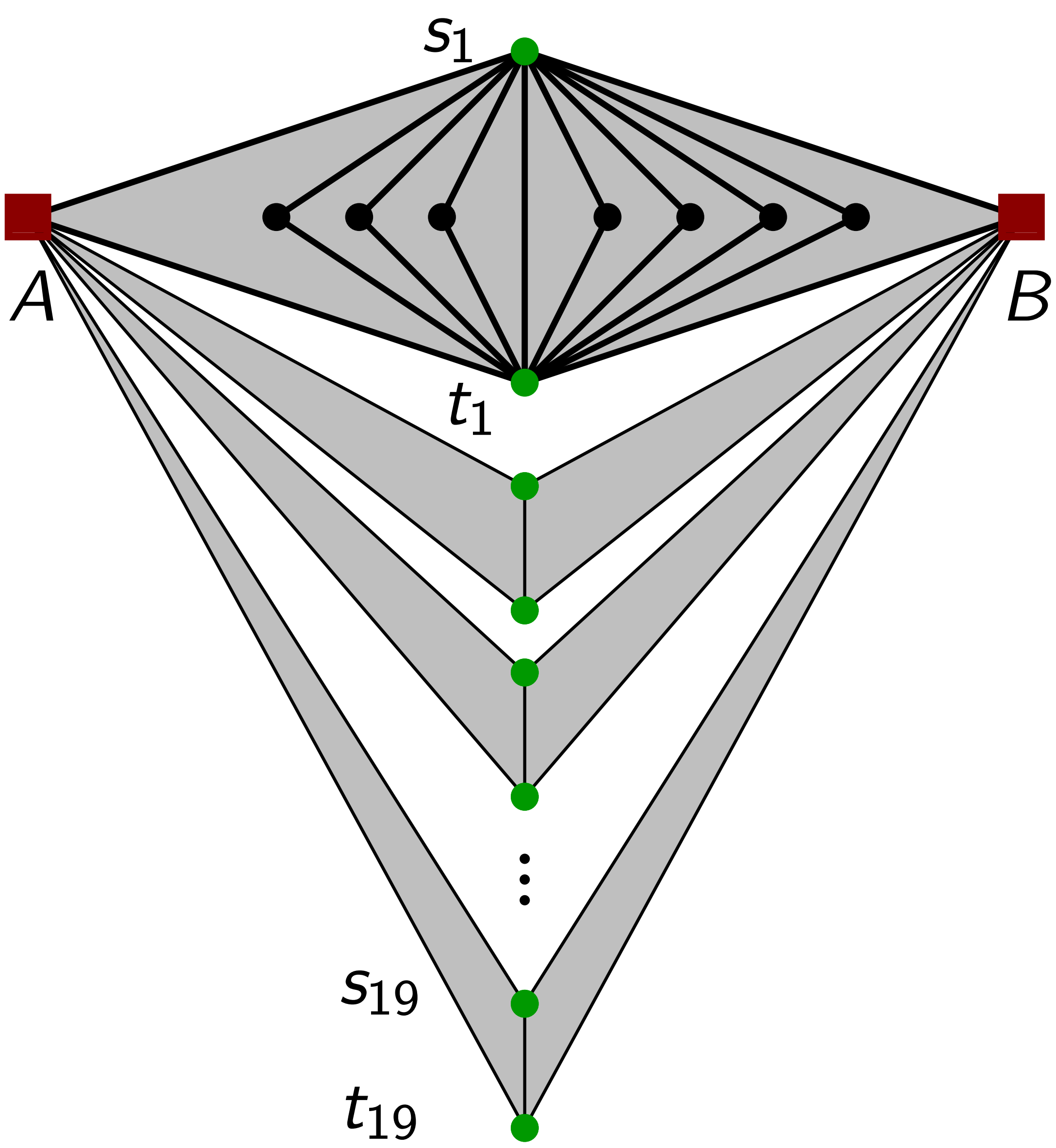


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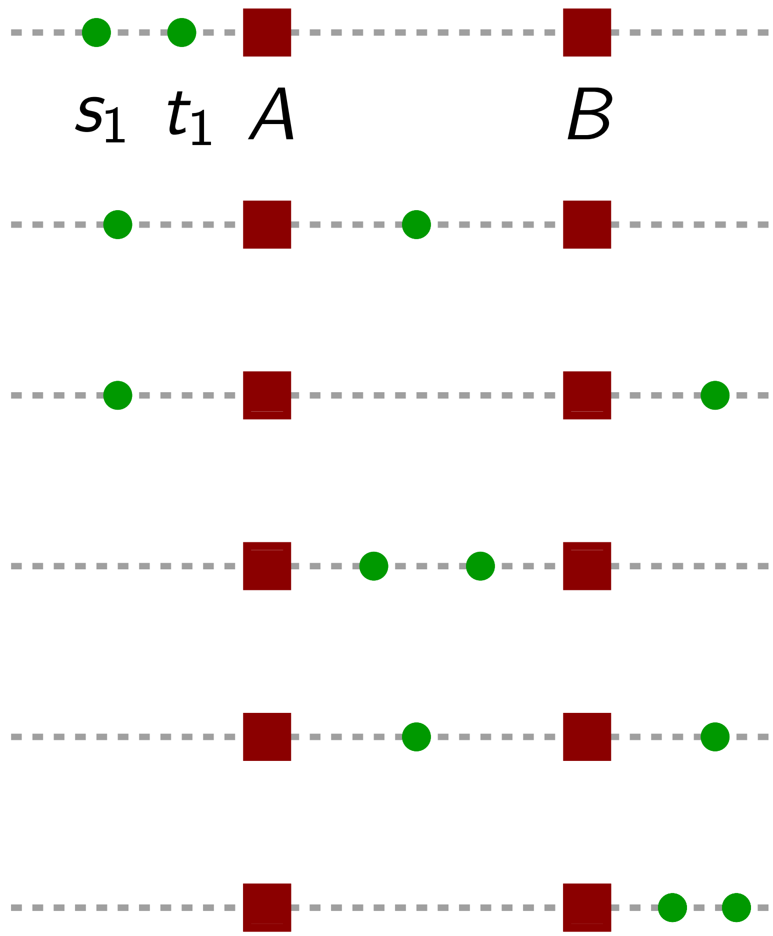

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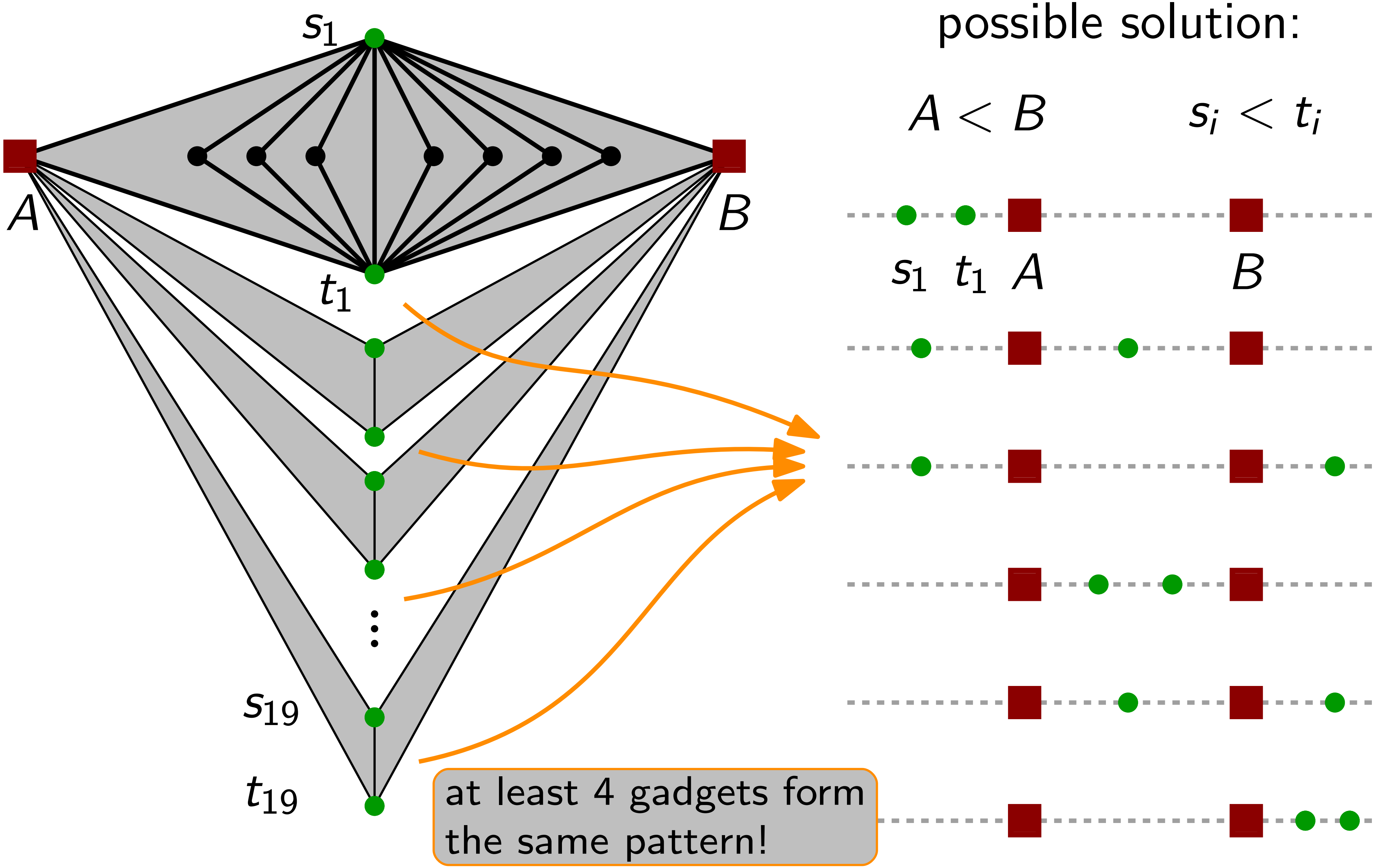

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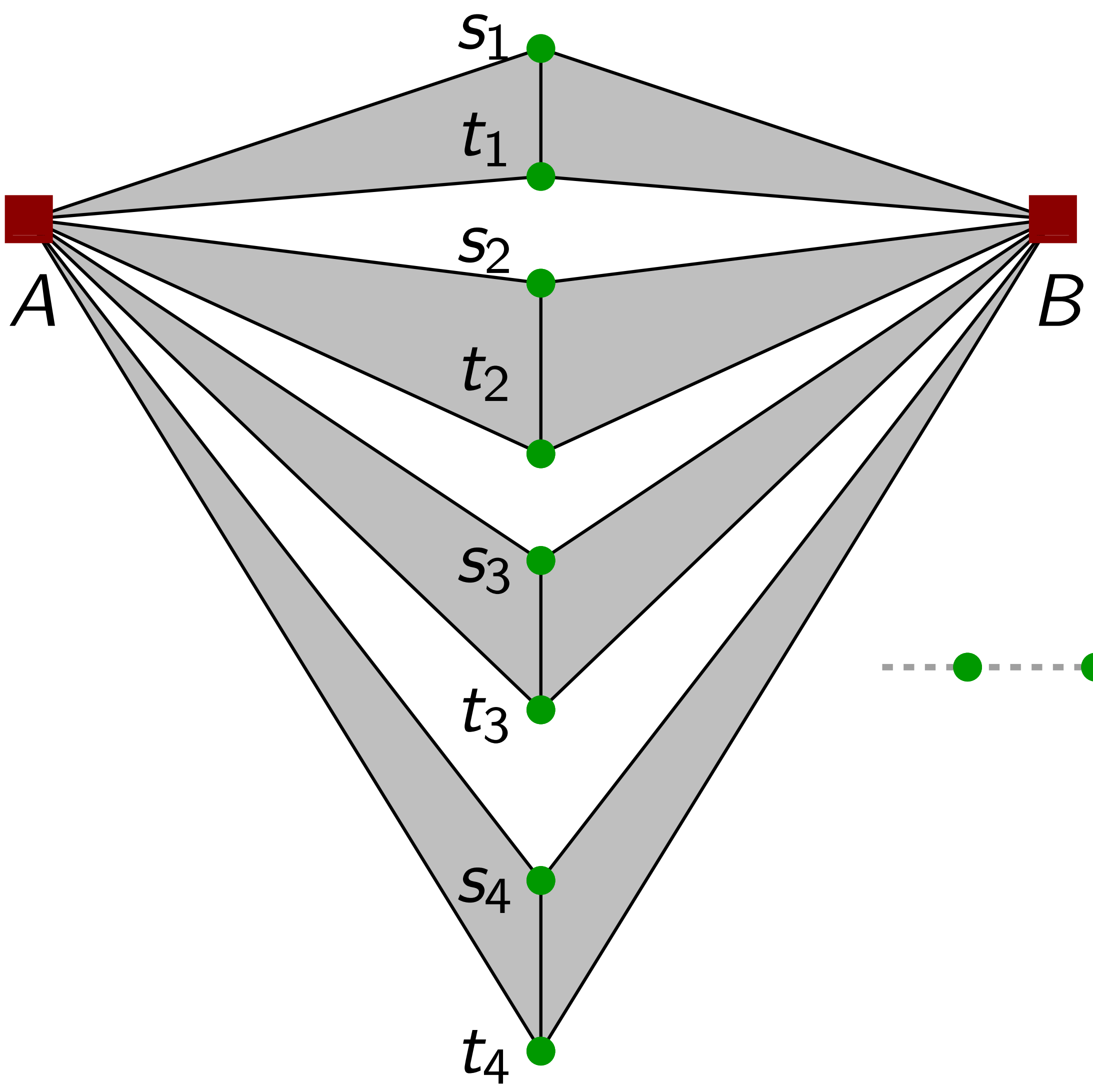


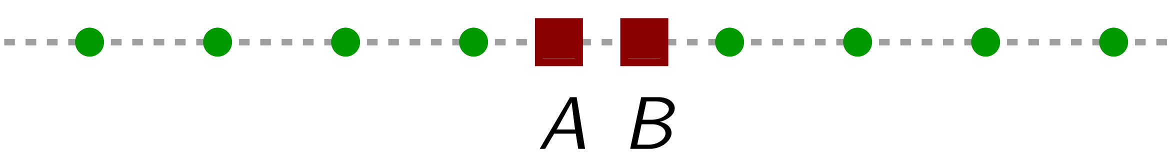

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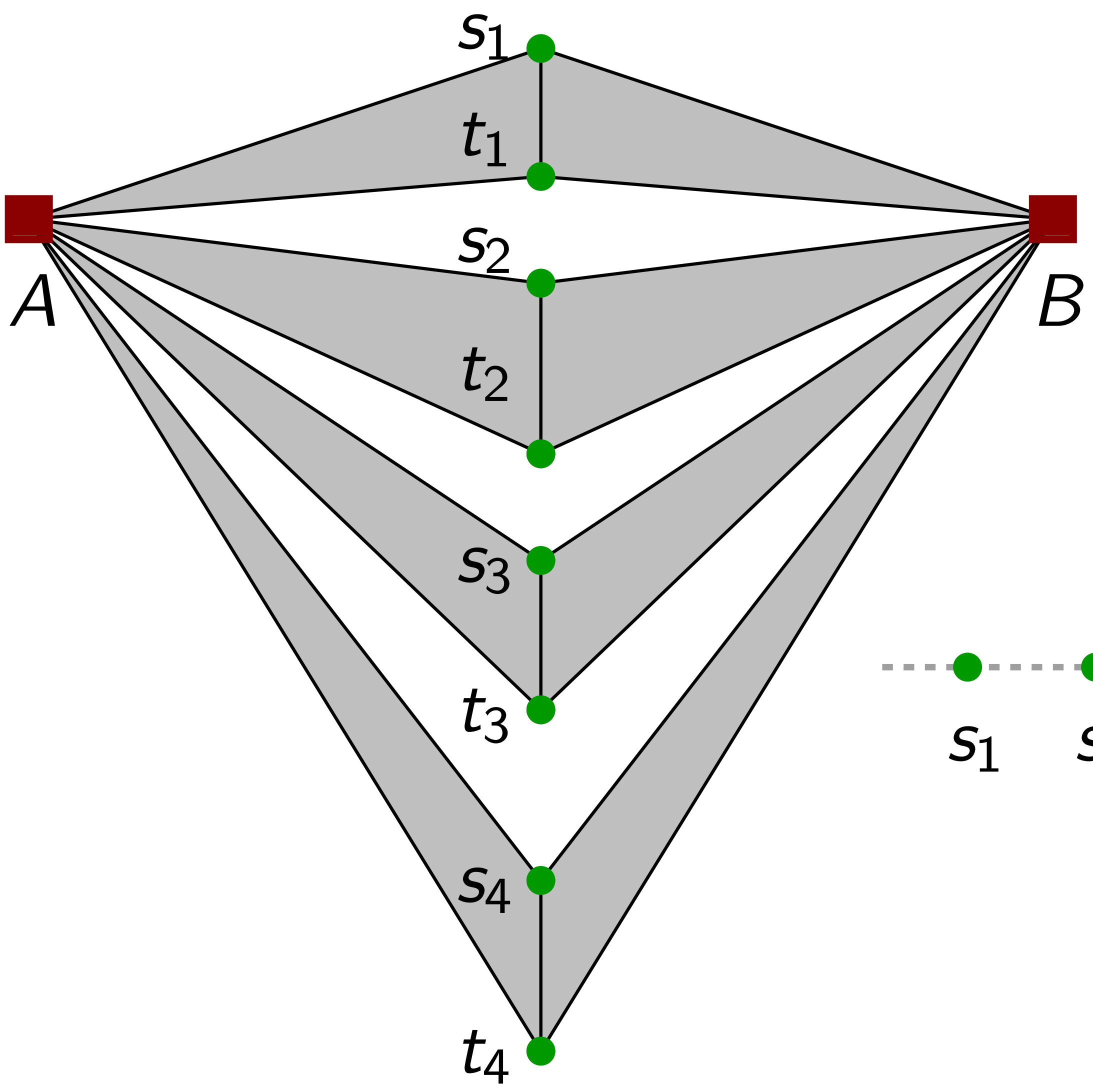


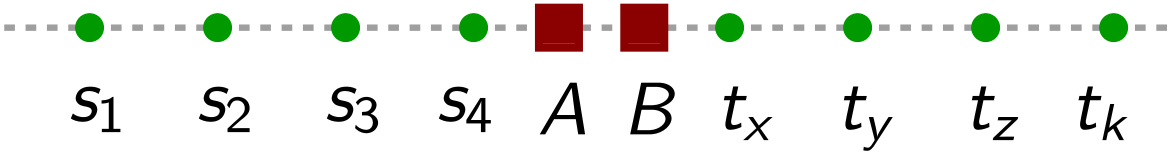

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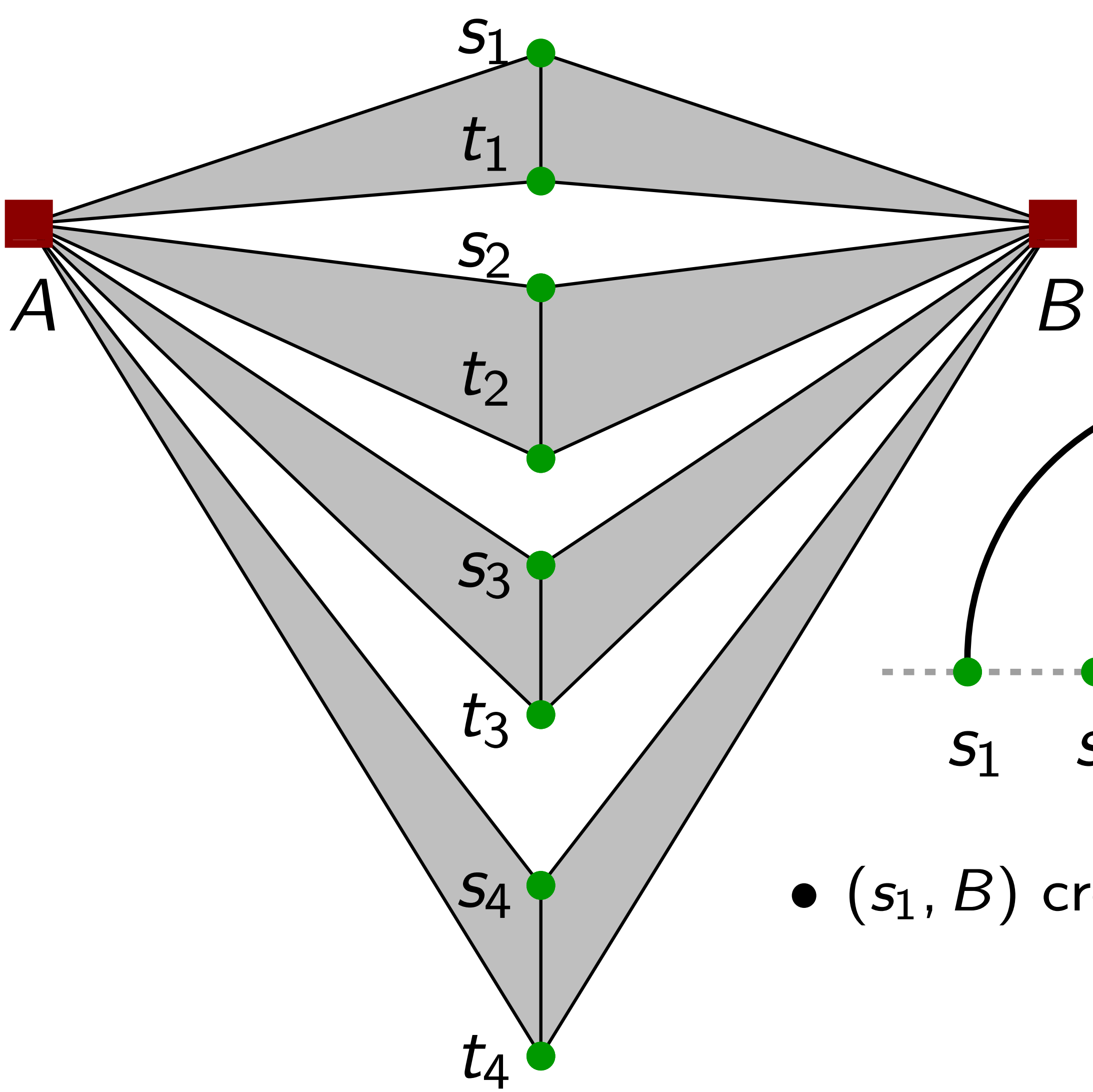


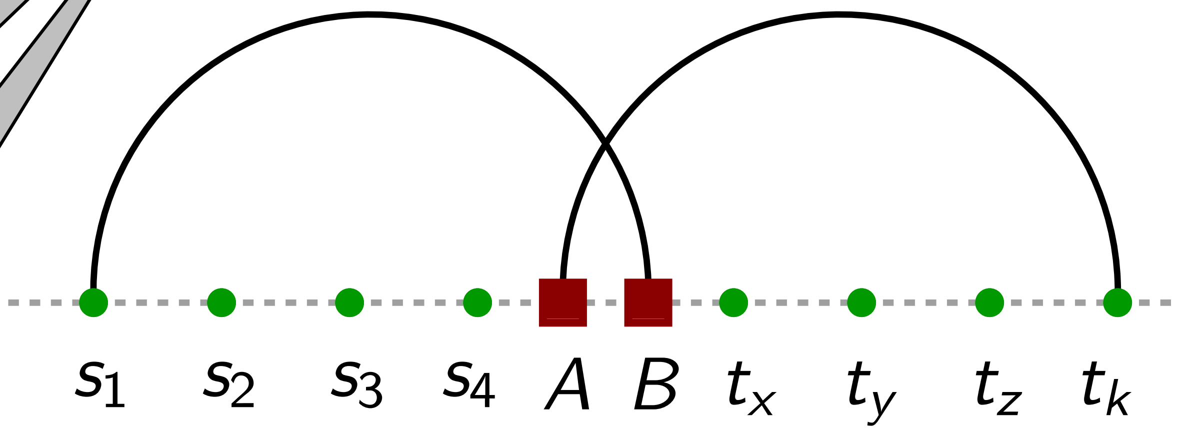


- $(s_1, B)$ crosses $(A, t_k)$

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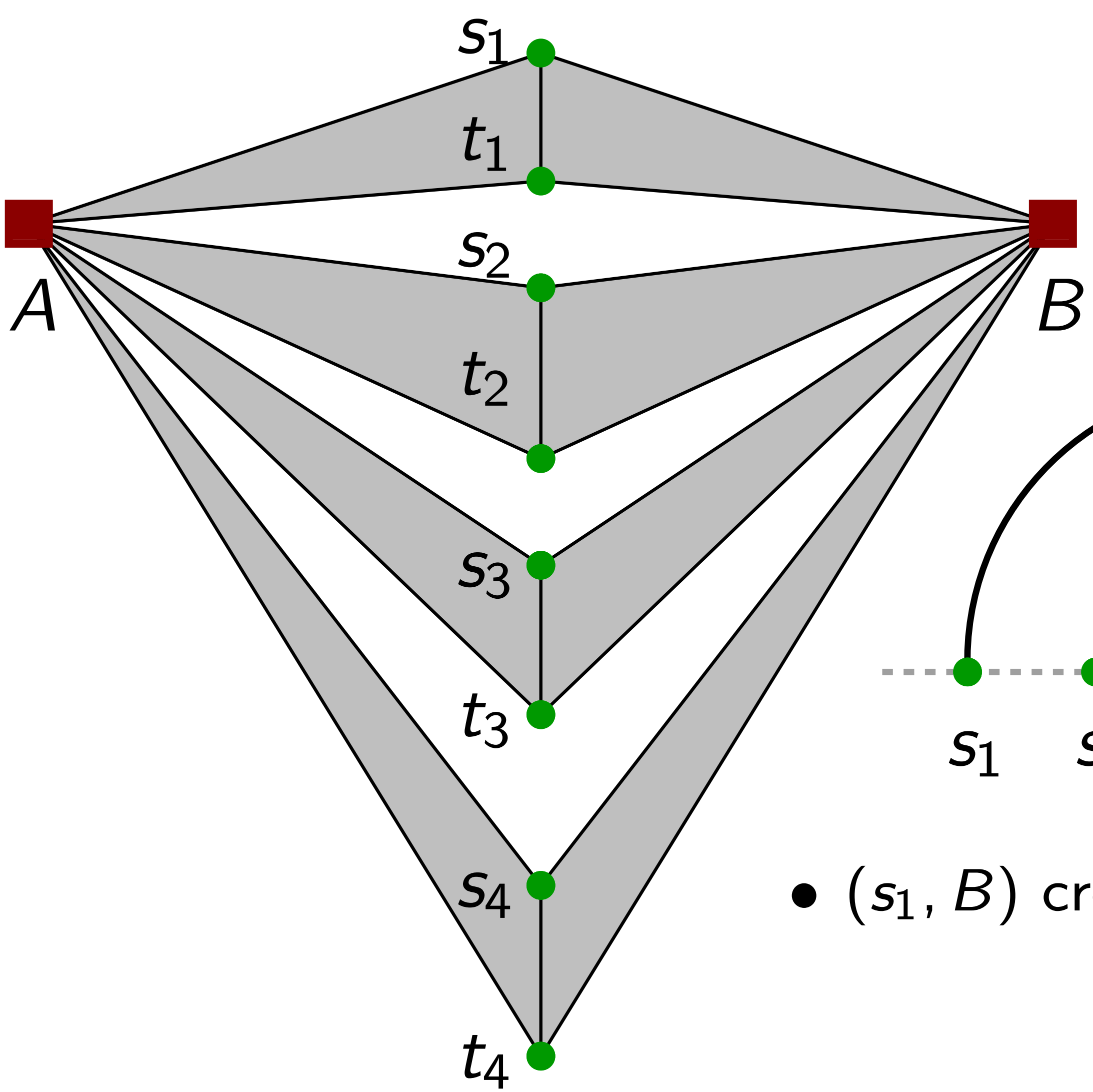


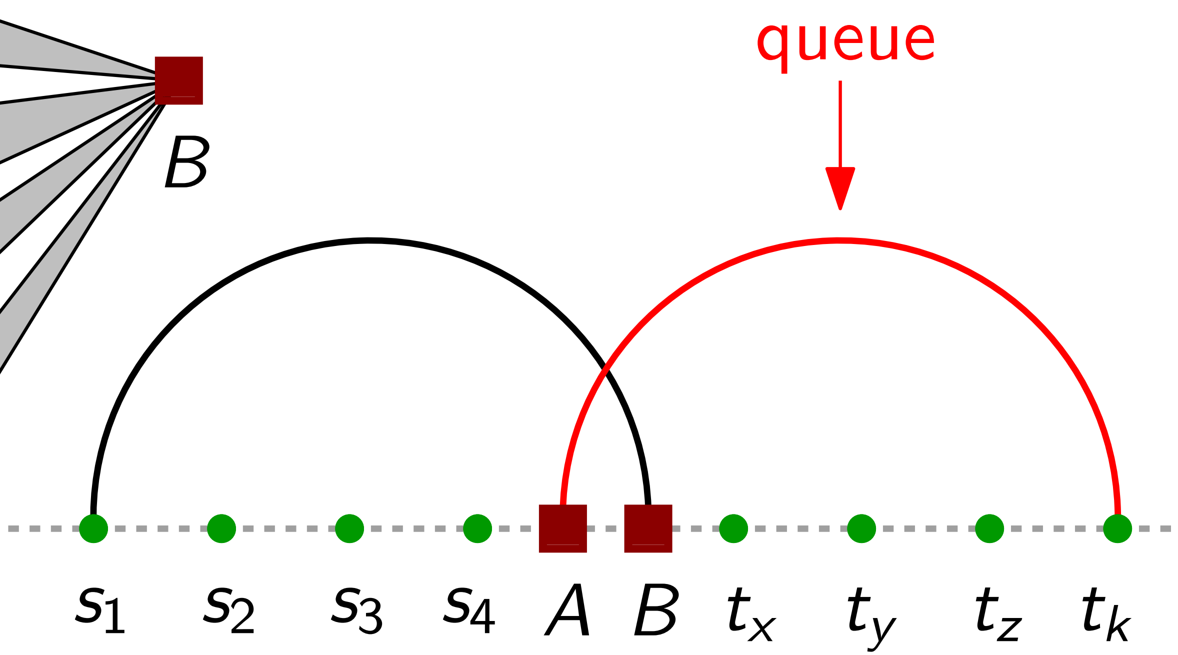


- $(s_1, B)$ crosses $(A, t_k)$ $\Rightarrow$wlog $(A, t_k) \in queue$

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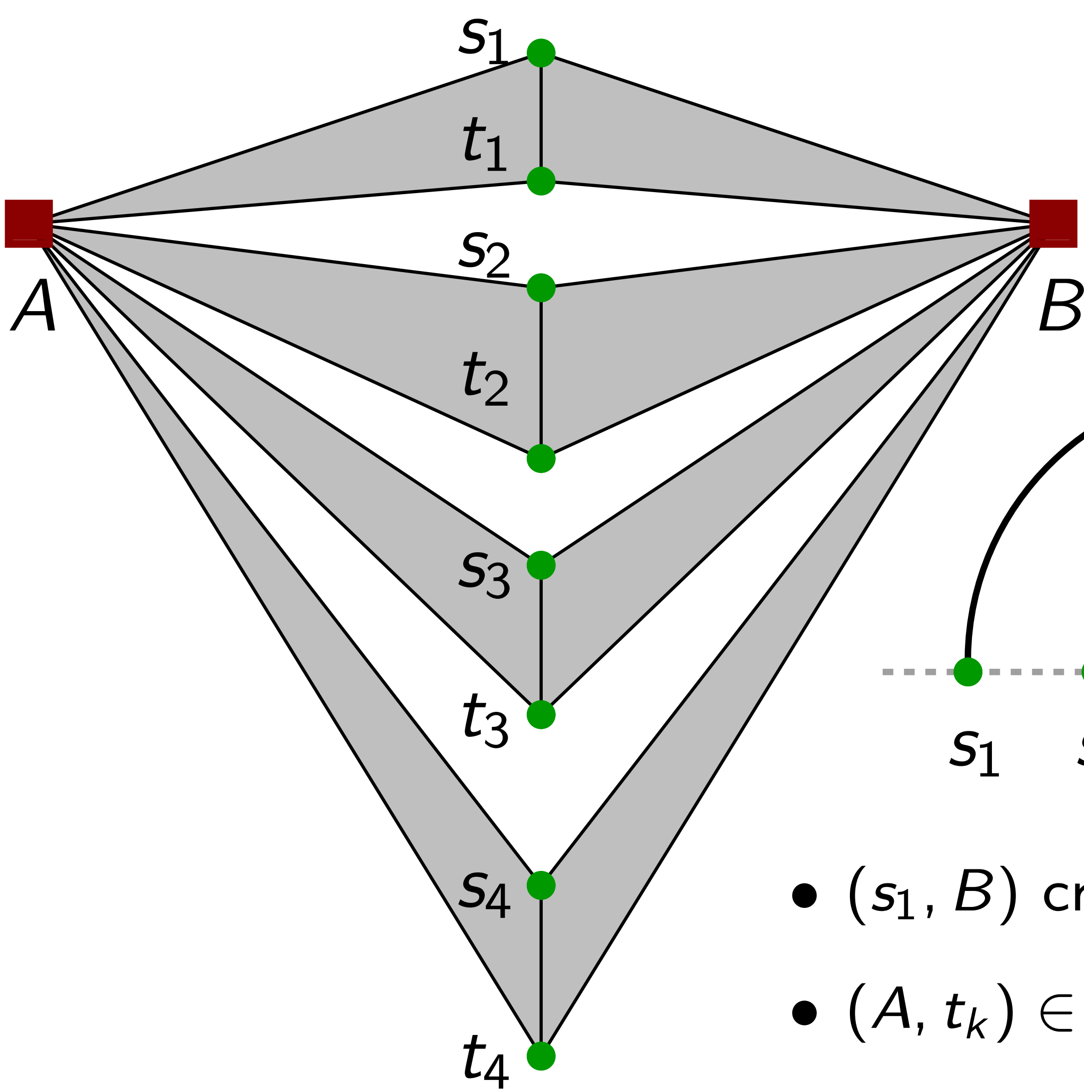


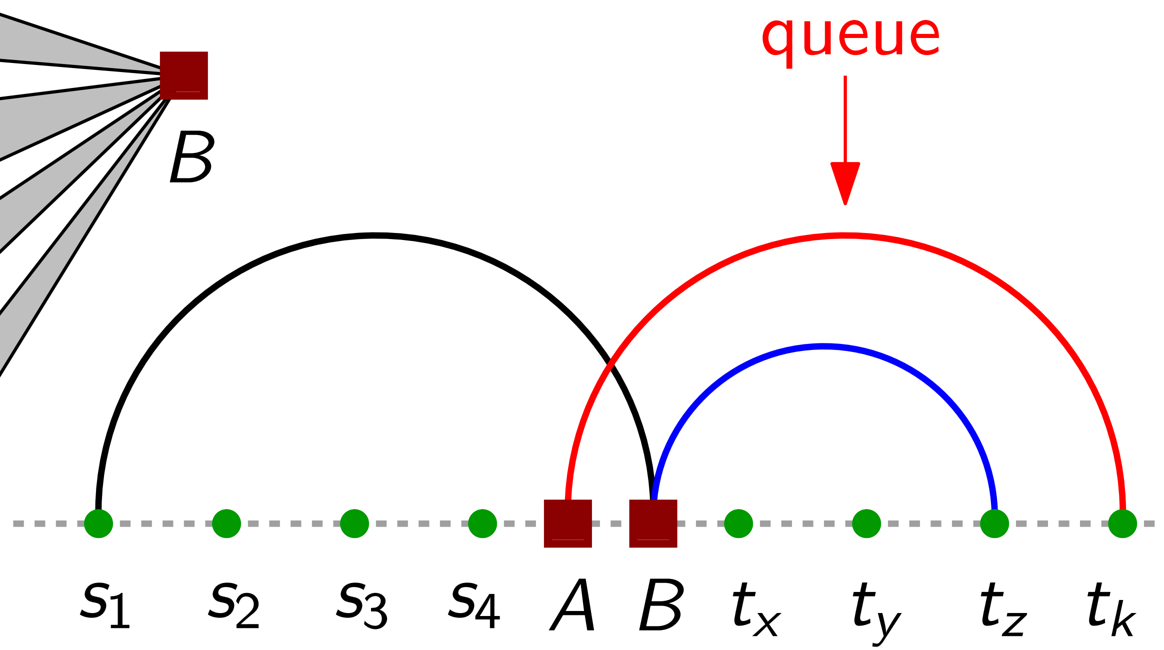


- $(s_1, B)$ crosses $(A, t_k)$ $\Rightarrow$wlog $(A, t_k) \in queue$
- $(A, t_k) \in queue$ $\Rightarrow$ $(B, t_z) \in stack$

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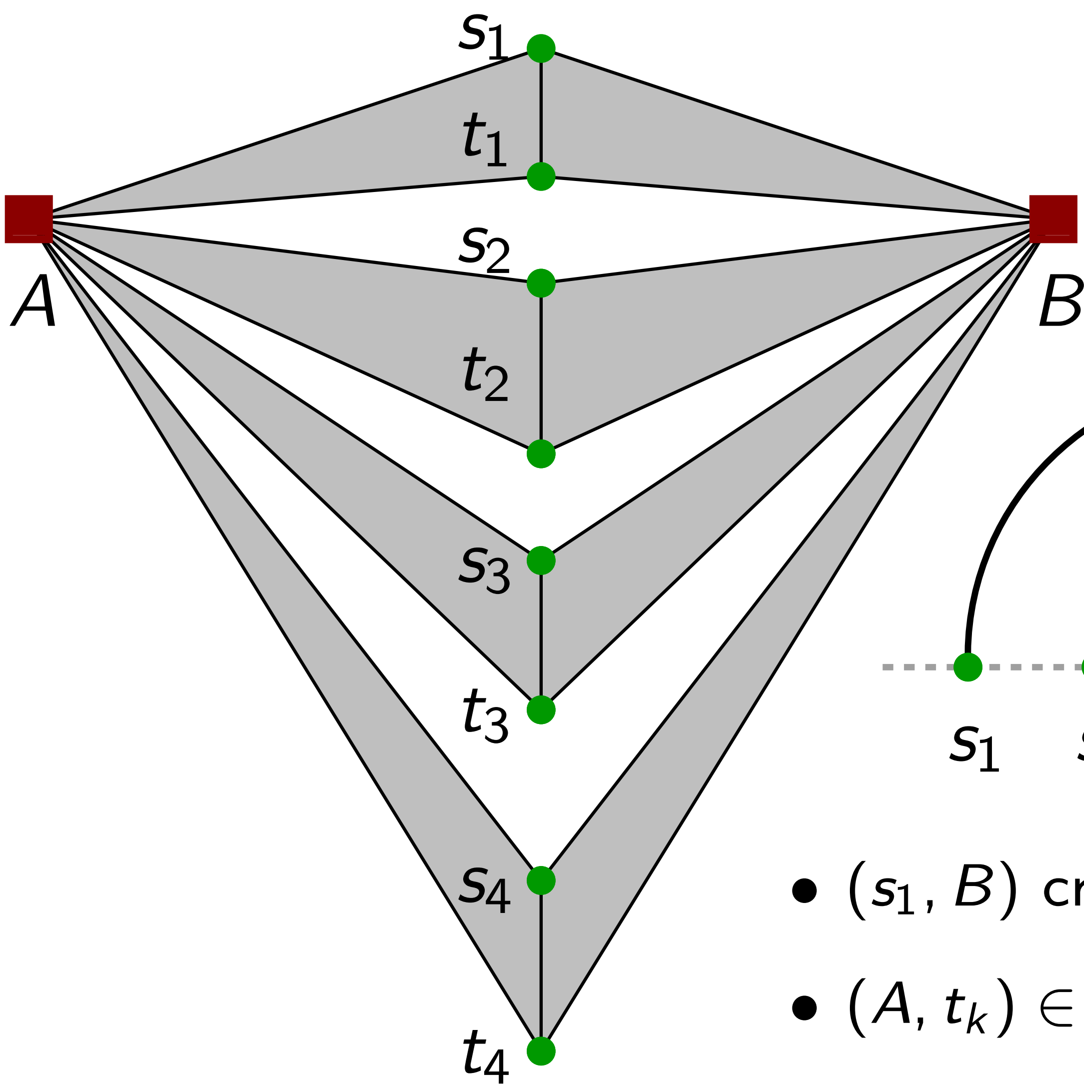


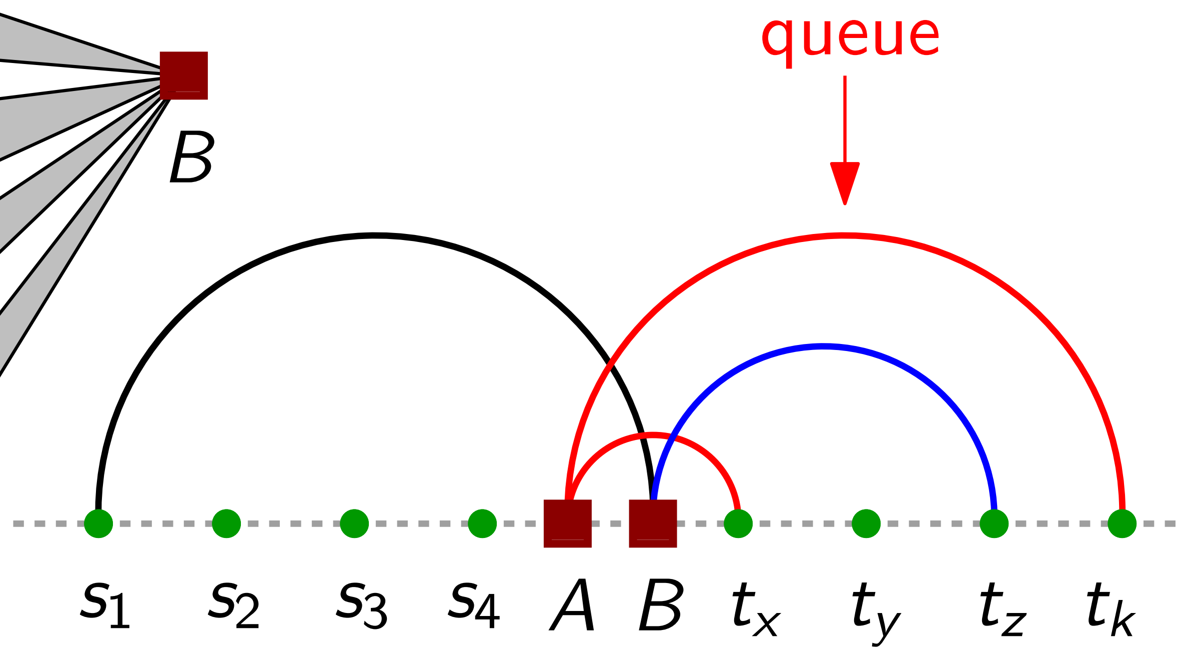


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- $(A, t_k) \in queue$ $\Rightarrow$ $(B, t_z) \in stack$
- $(s_1, B) \in stack$ $\Rightarrow$ $(A, t_x) \in queue$

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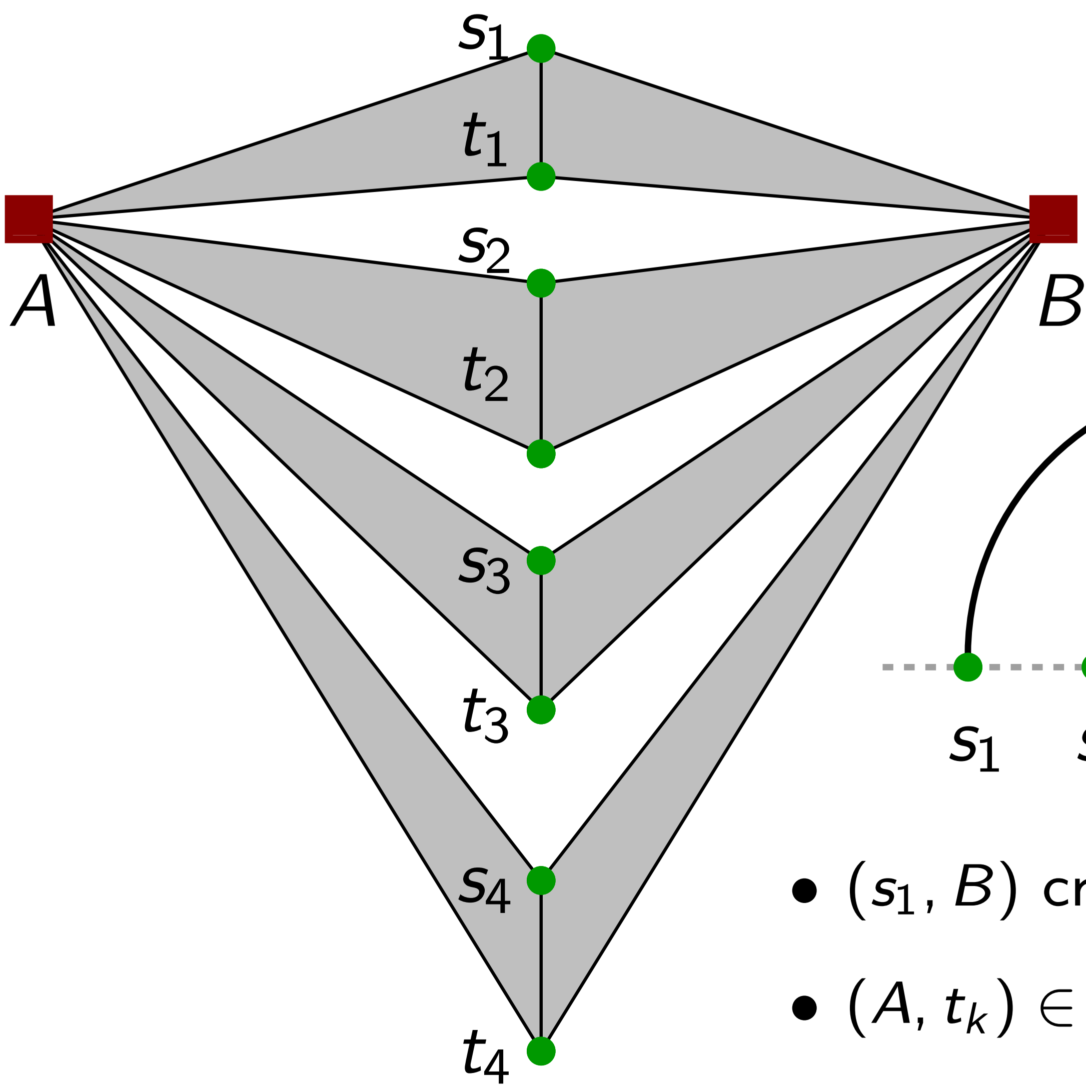


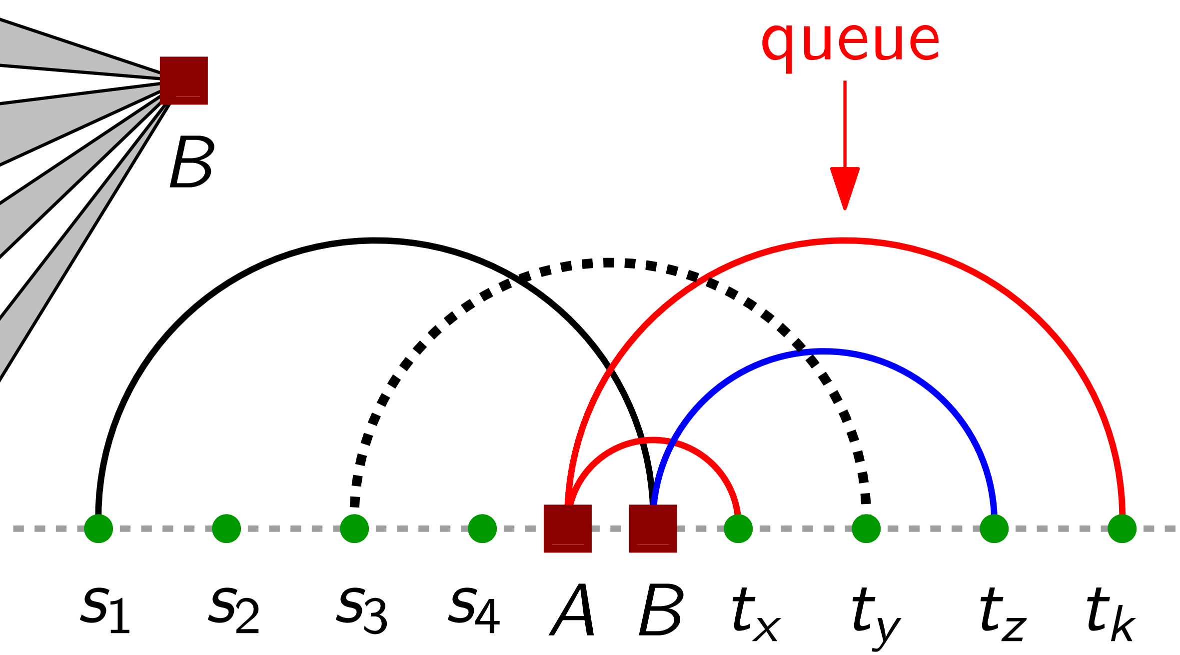


- $(s_1, B)$ crosses $(A, t_k)$ $\Rightarrow$wlog $(A, t_k) \in queue$
- $(A, t_k) \in queue$ $\Rightarrow$ $(B, t_z) \in stack$
- $(s_1, B) \in stack$ $\Rightarrow$ $(A, t_x) \in queue$
- cannot assign $(s, t_y)$: crosses $(B, t_z)$, covers $(A, t_x)$

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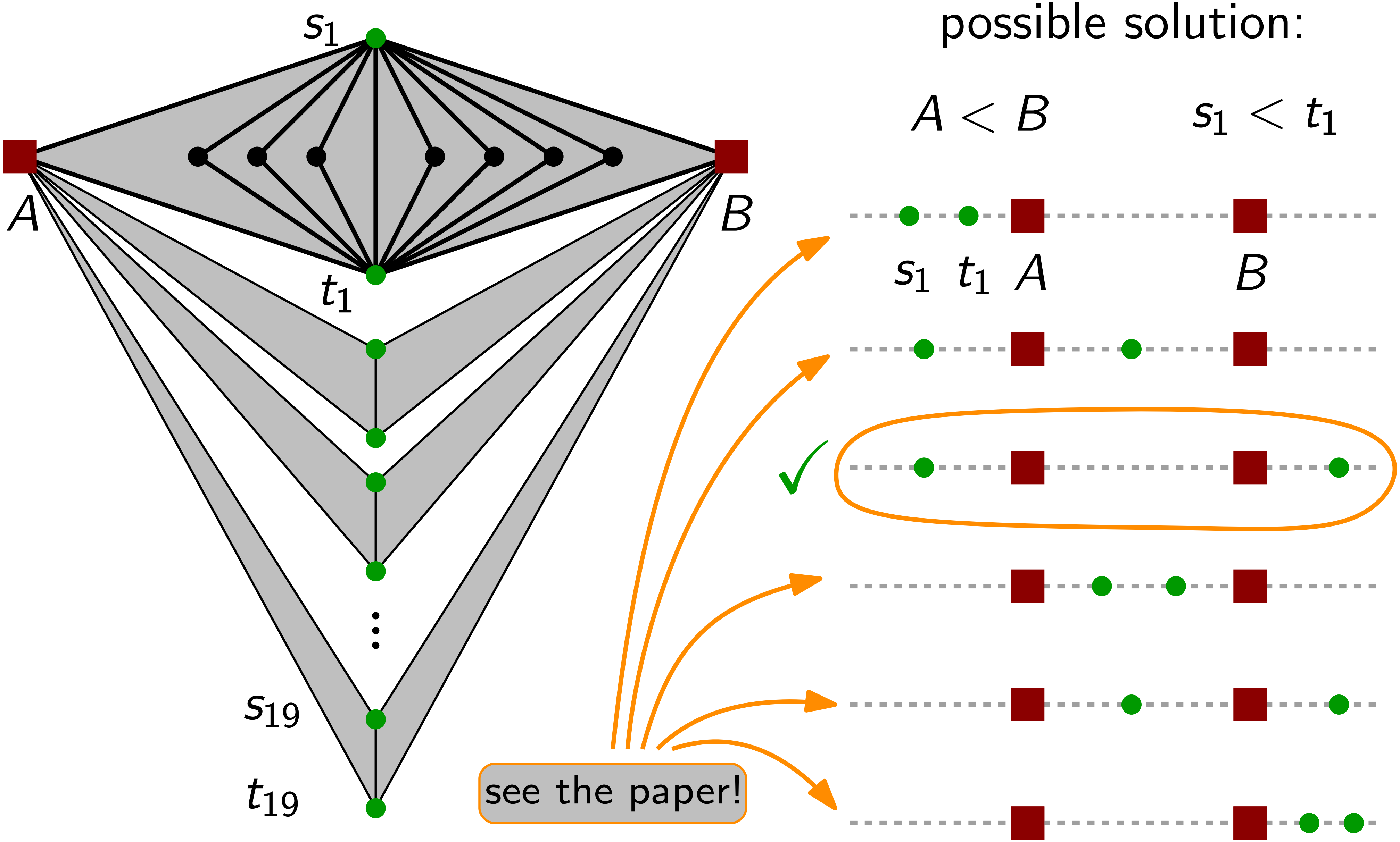

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**Theorem 2**

Every planar graph **admits** a mixed 1-stack 1-queue **subdivision** with one division vertex per edge

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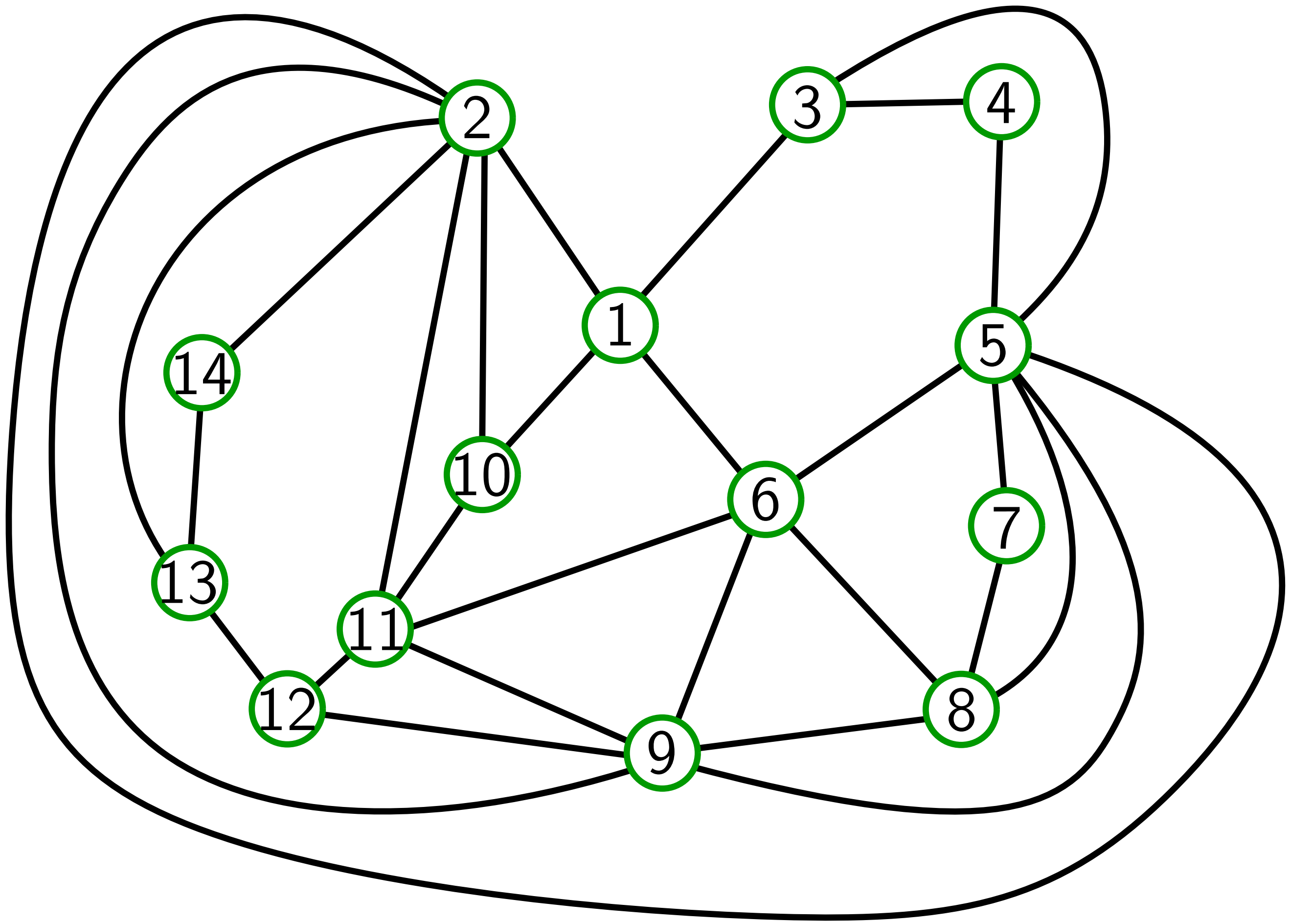

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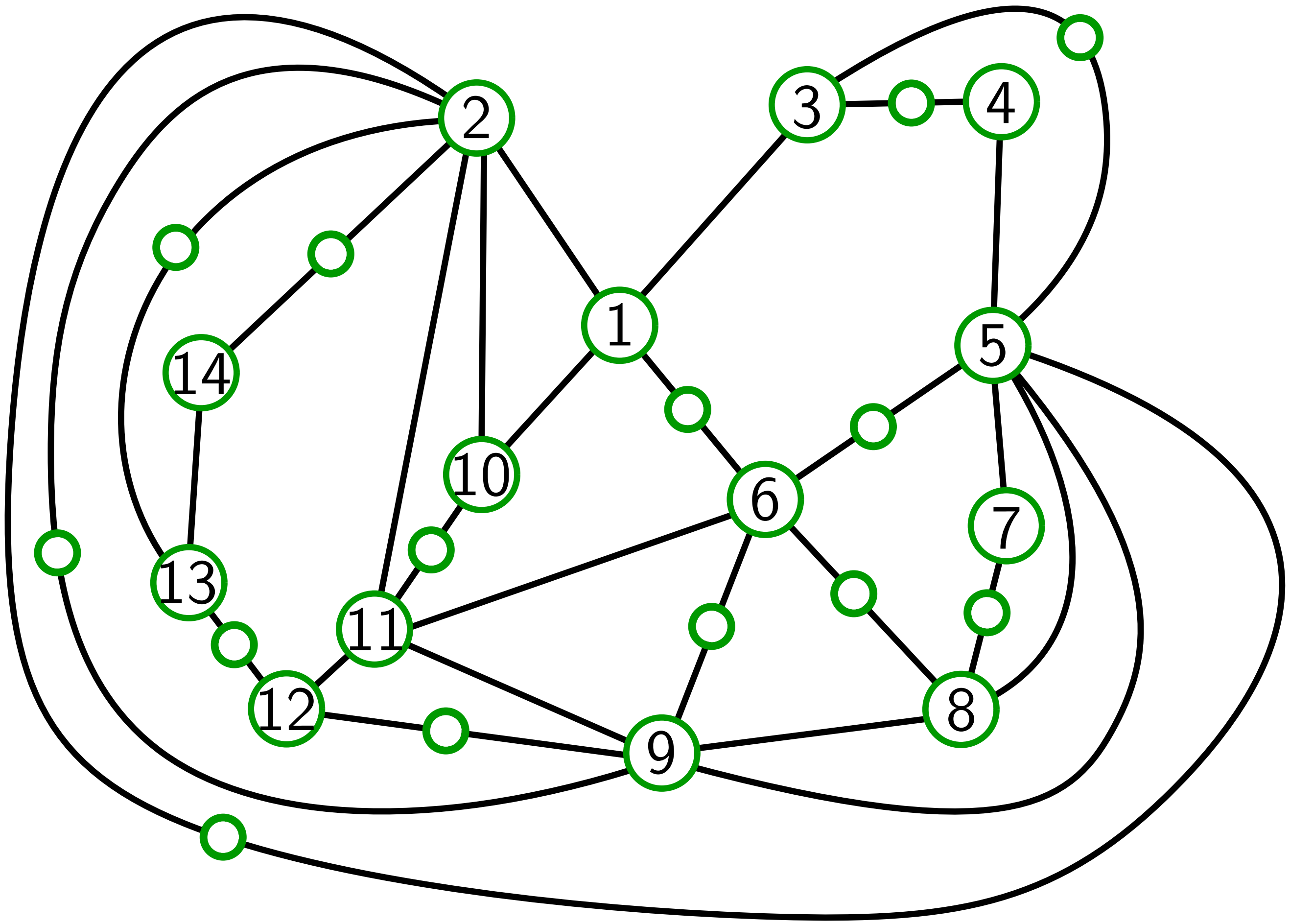

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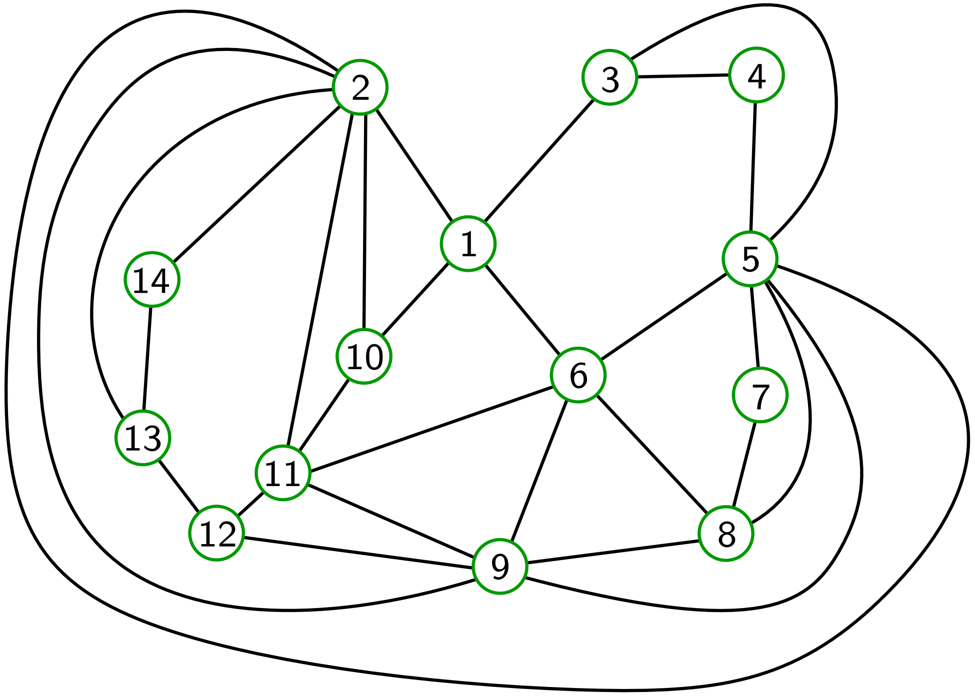

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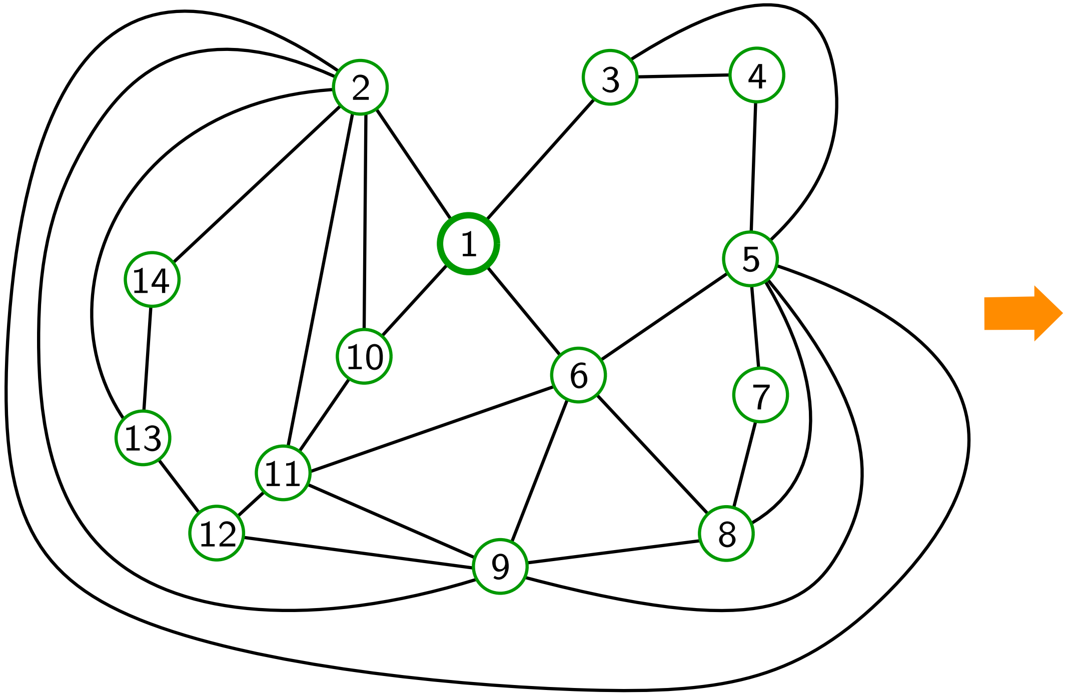


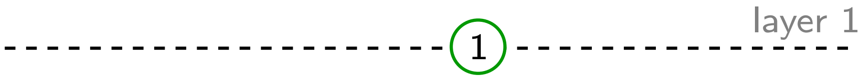

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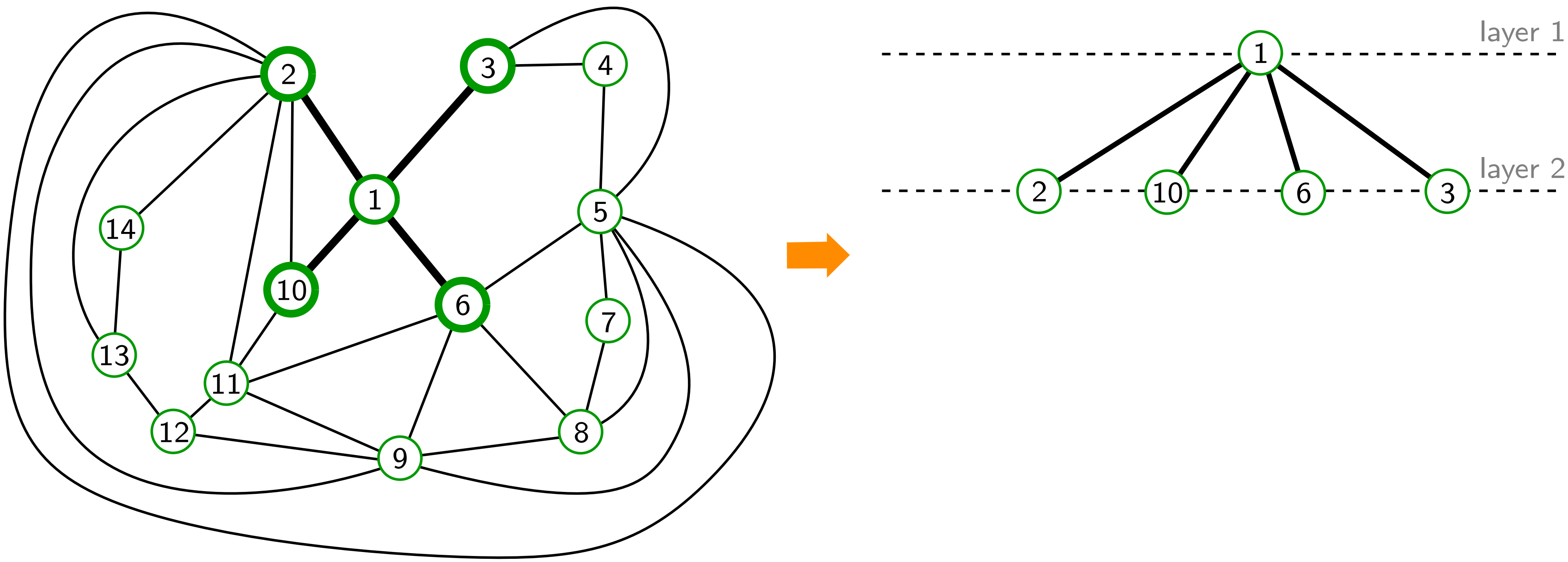

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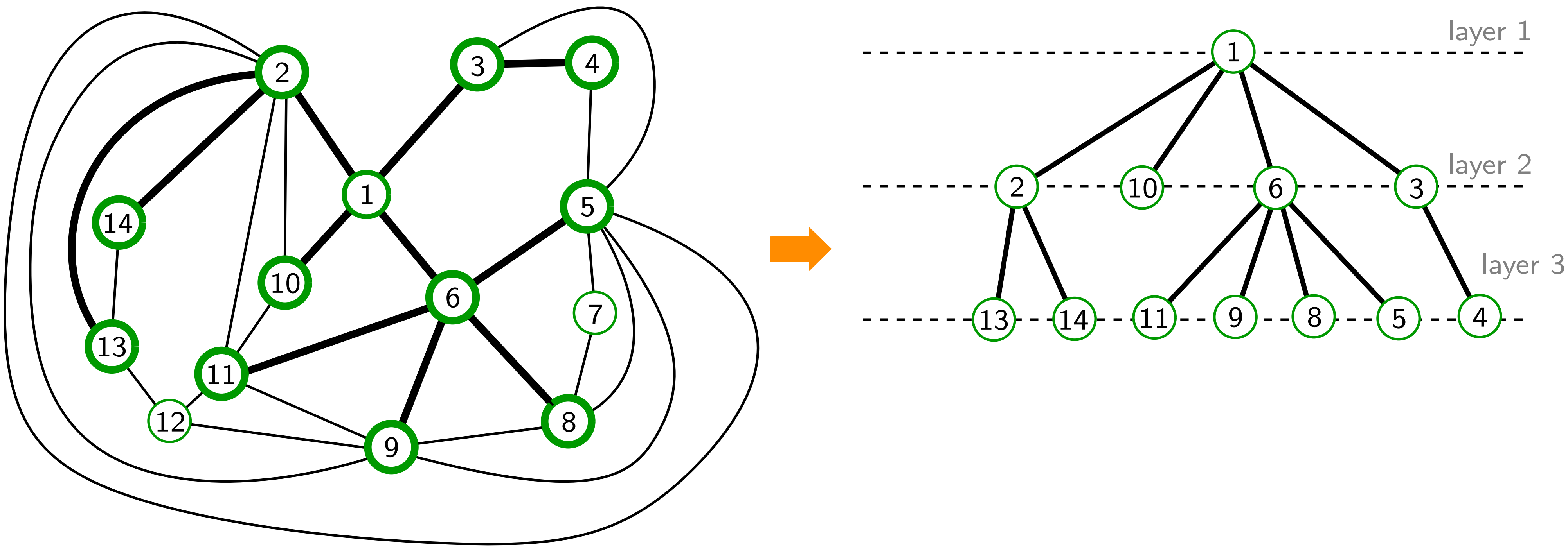

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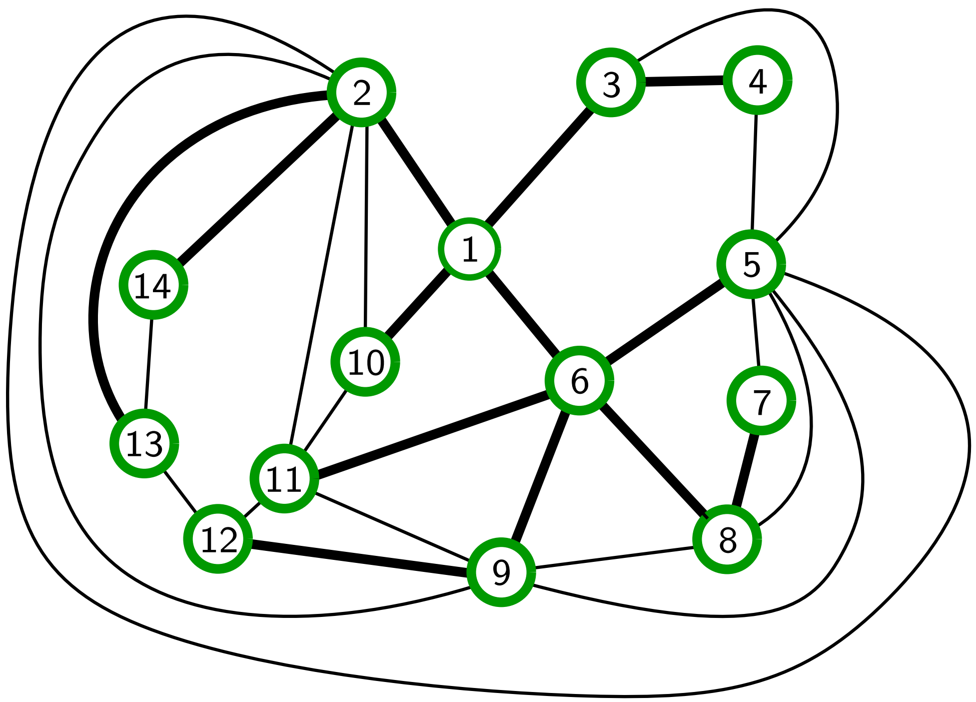


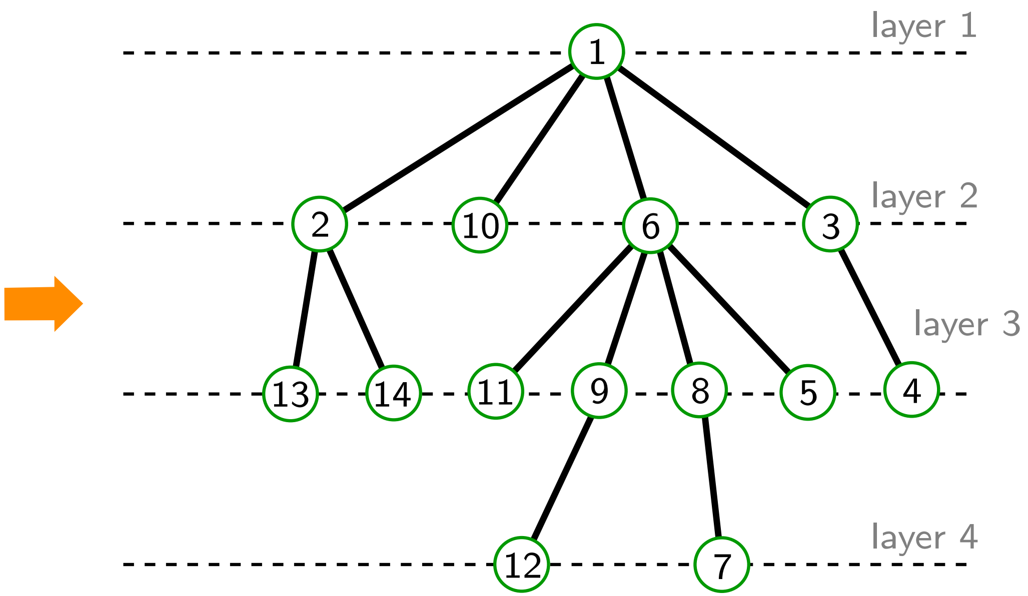

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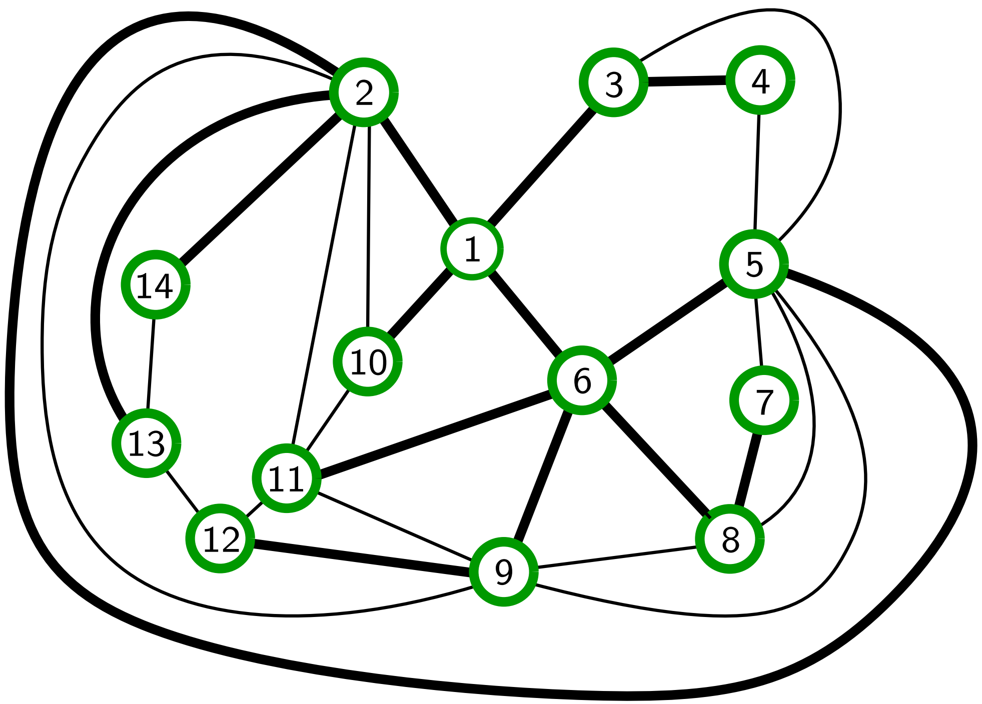


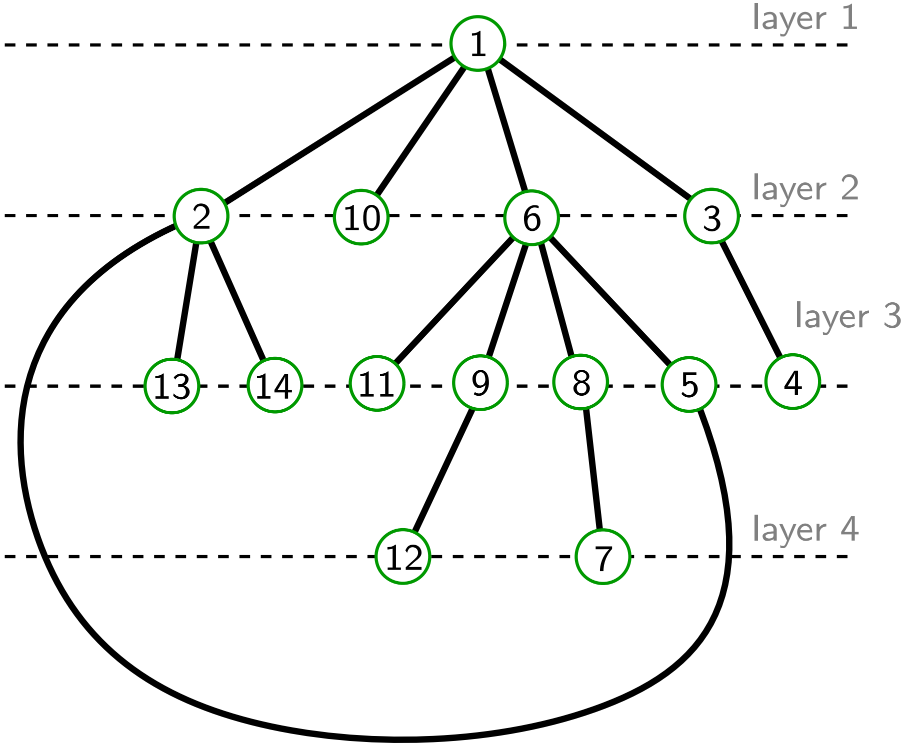

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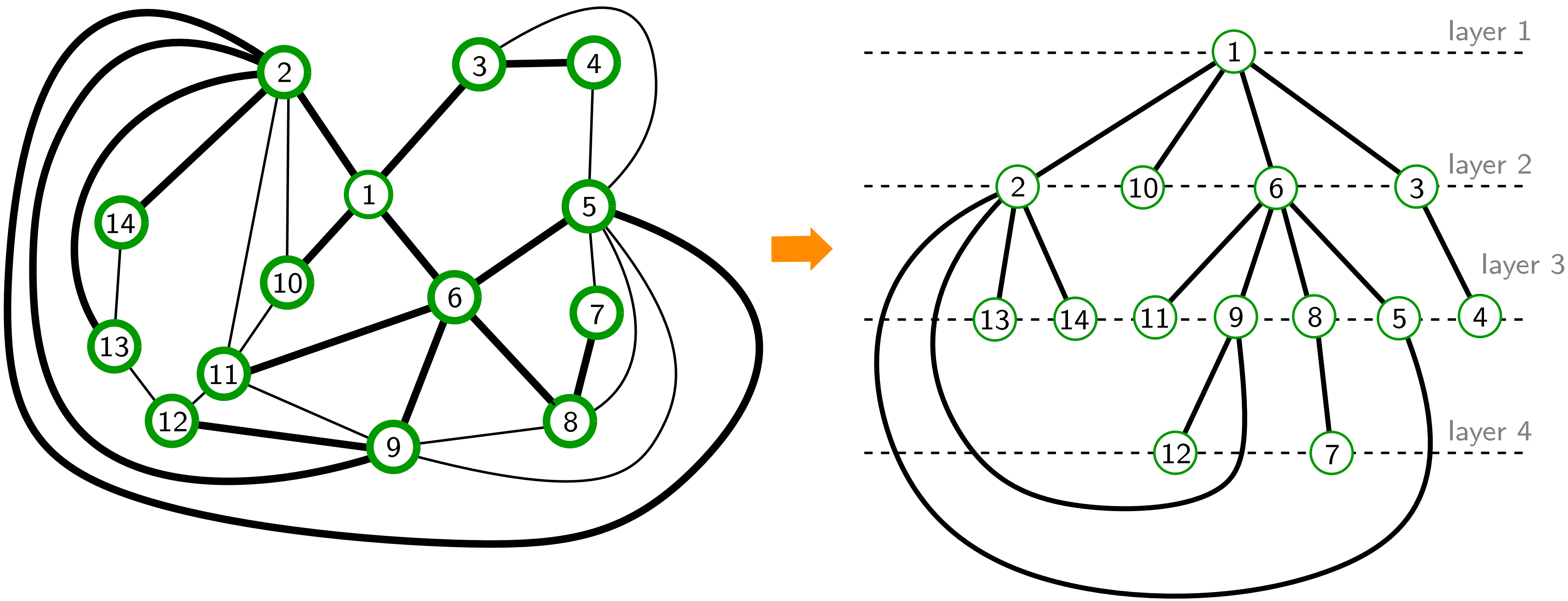

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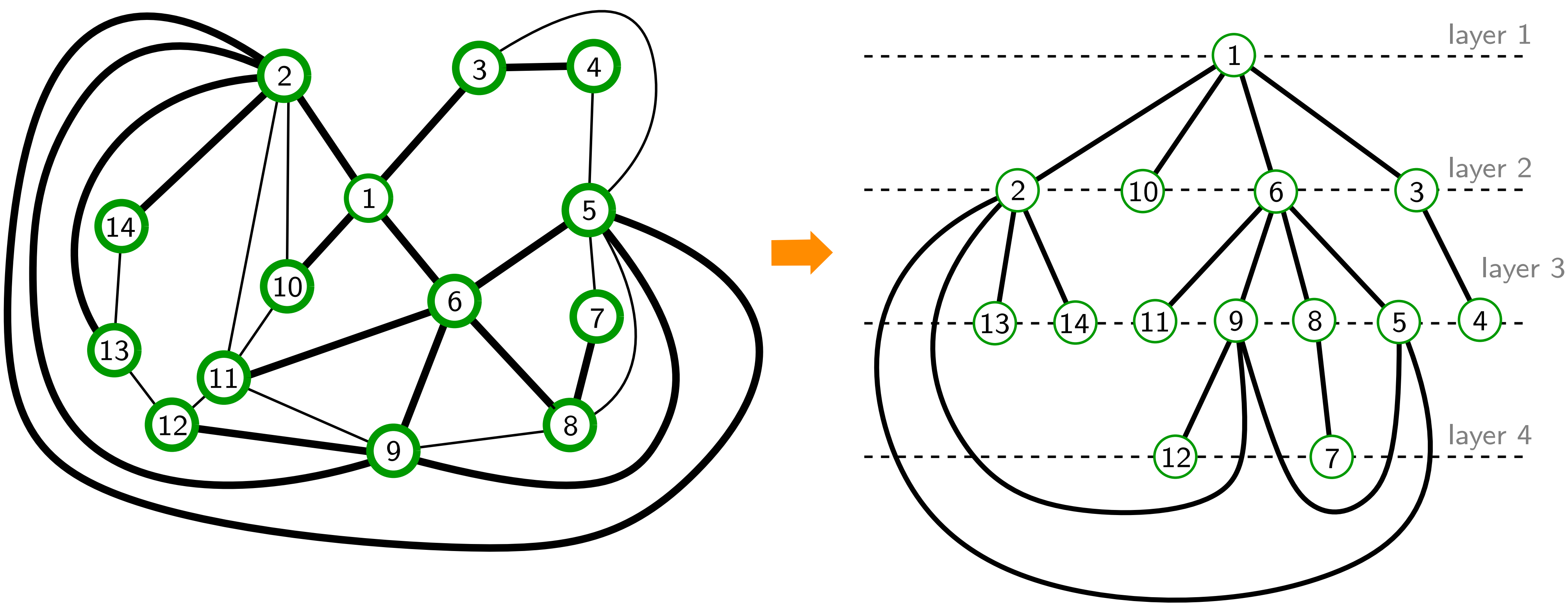

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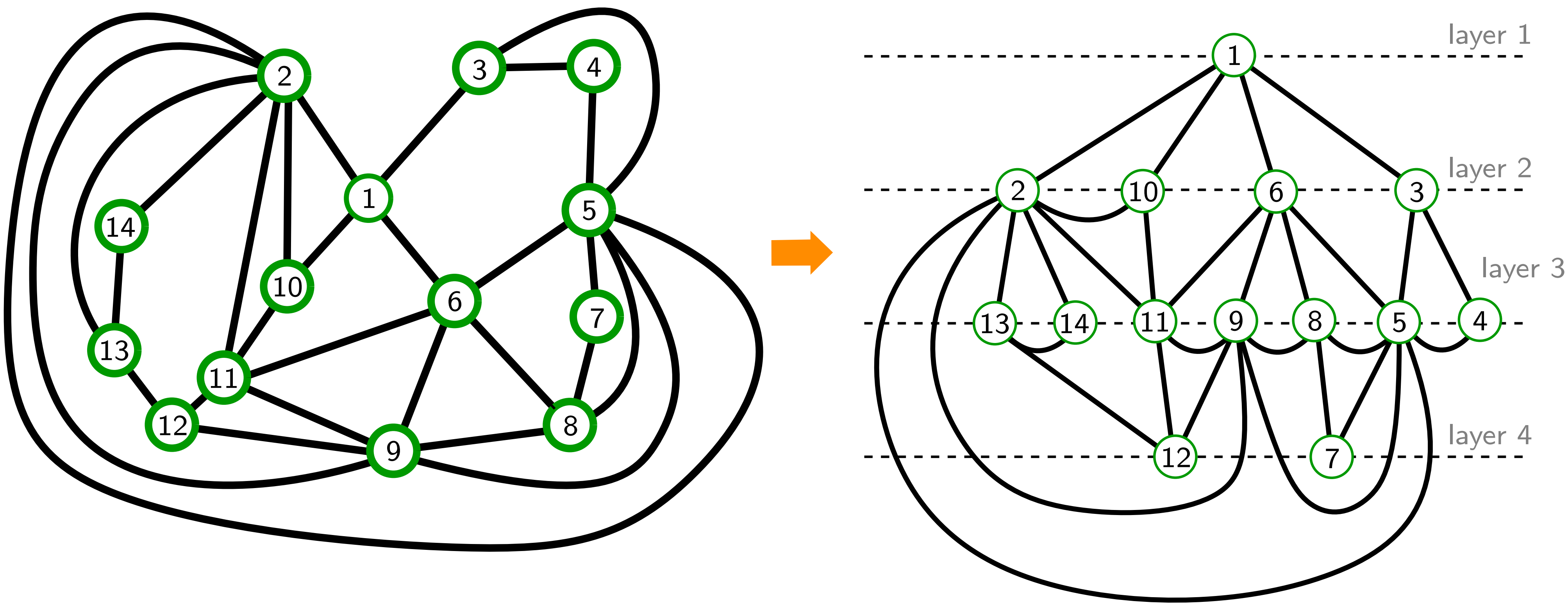

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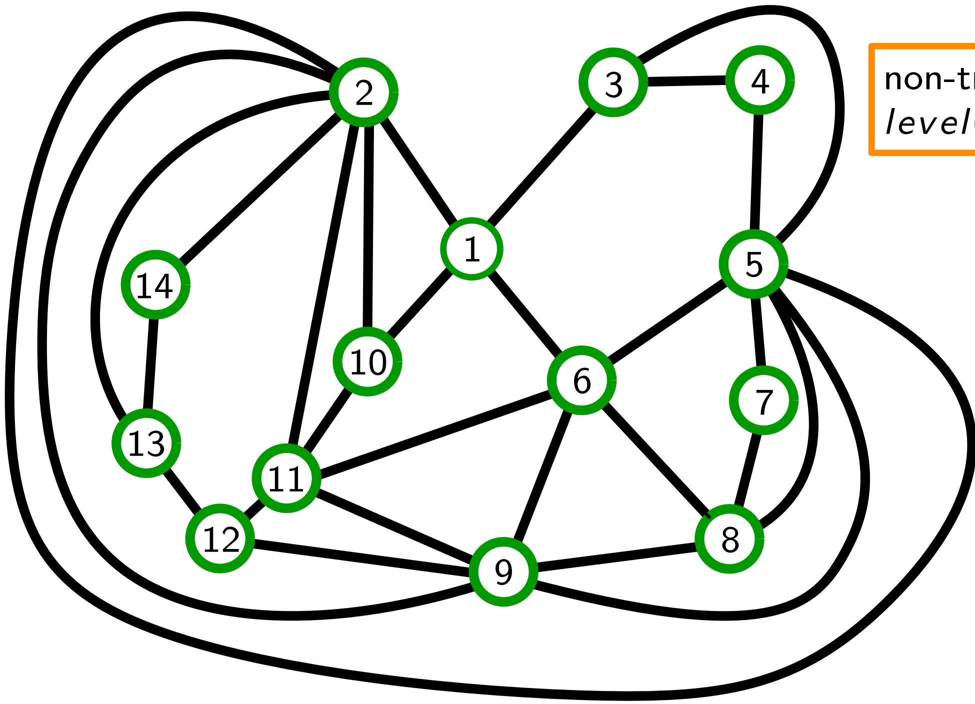


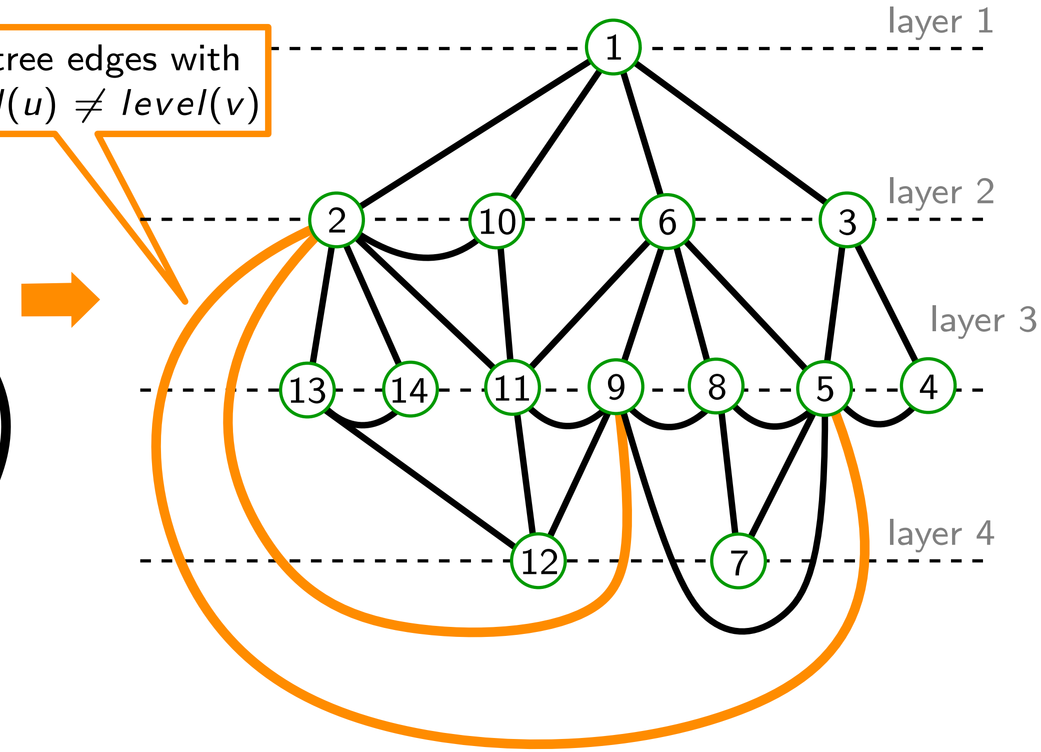

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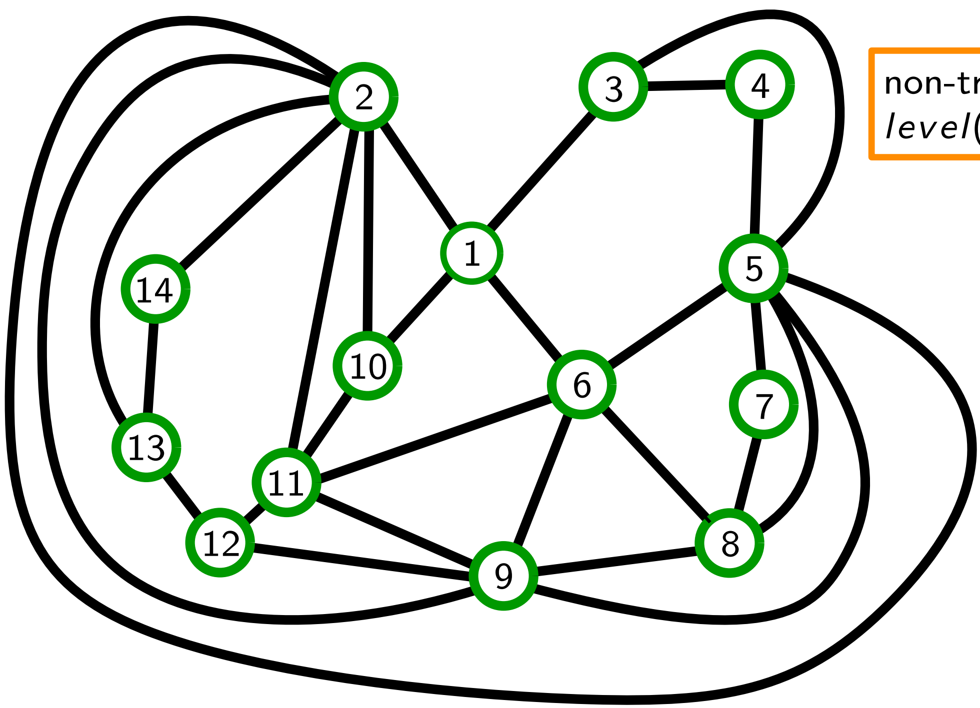


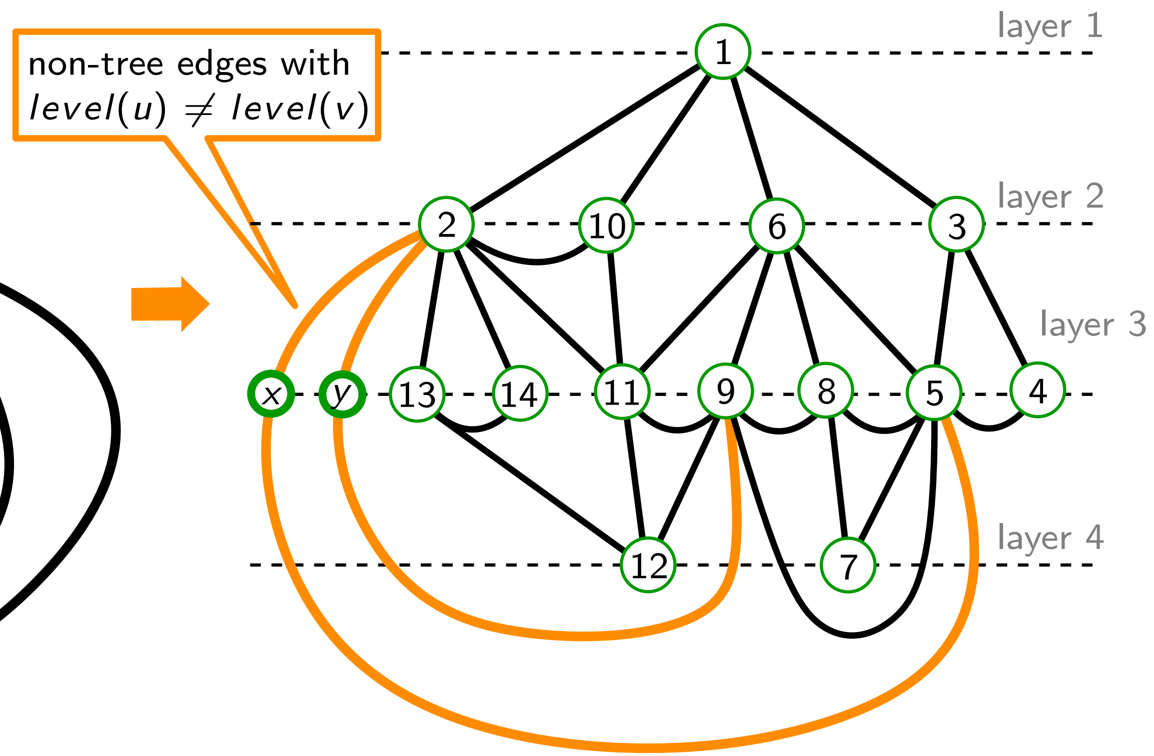

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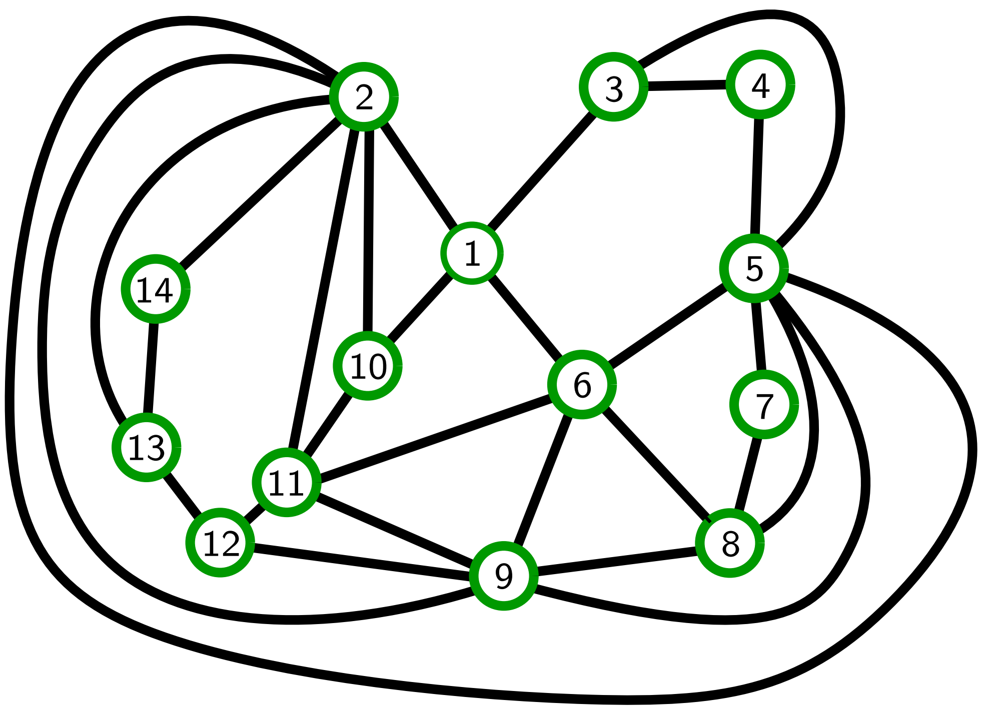


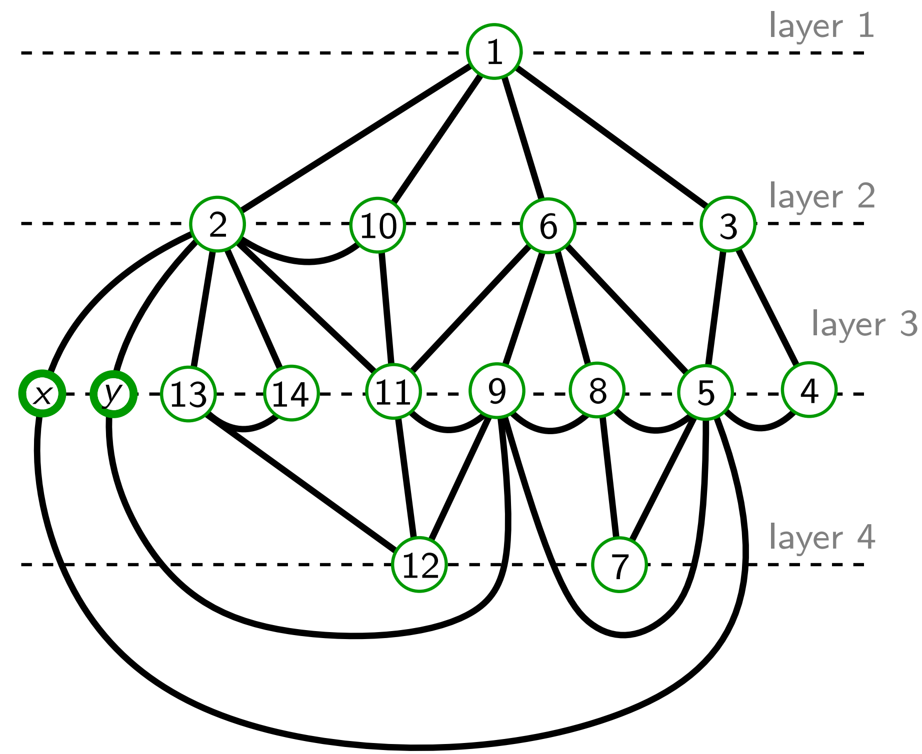


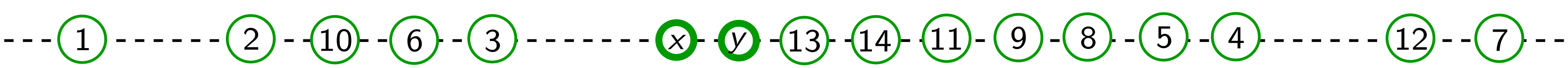

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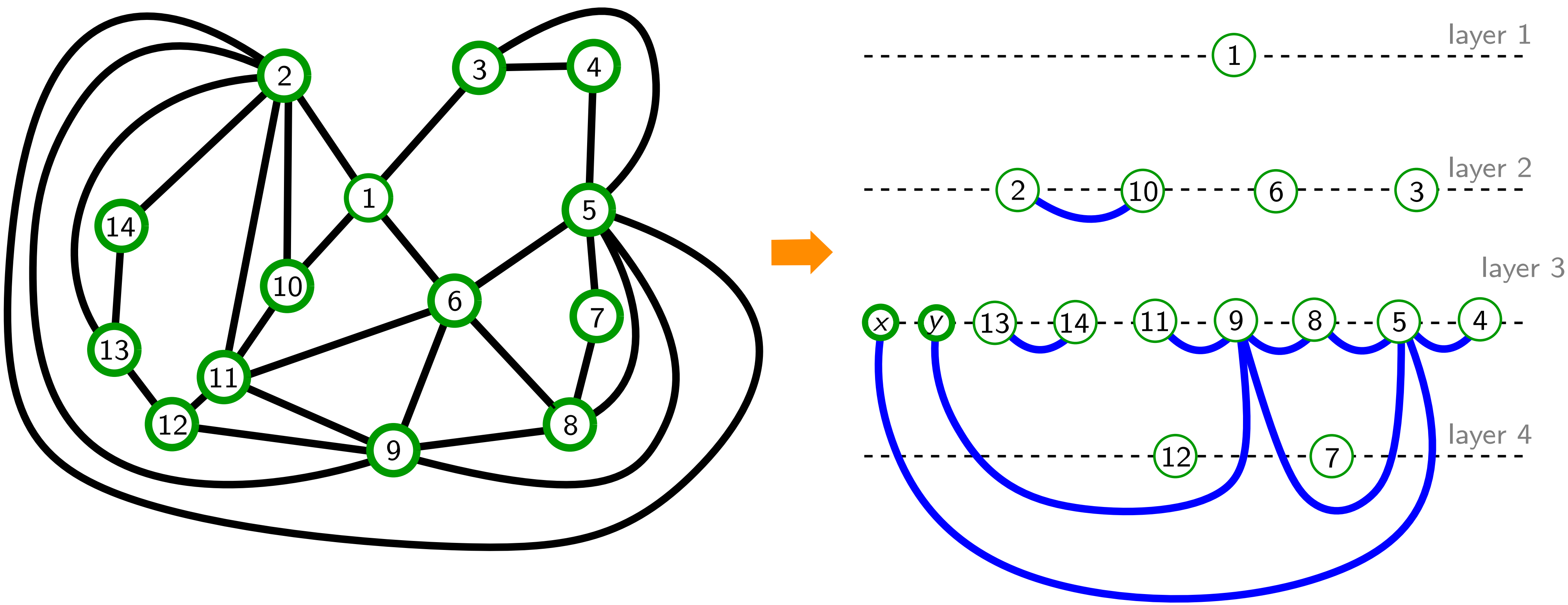


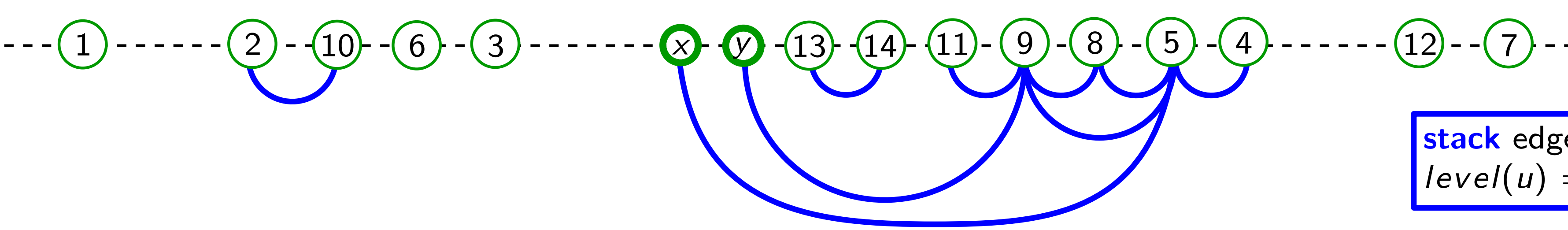


**stack** edges:
$level(u) = level(v)$

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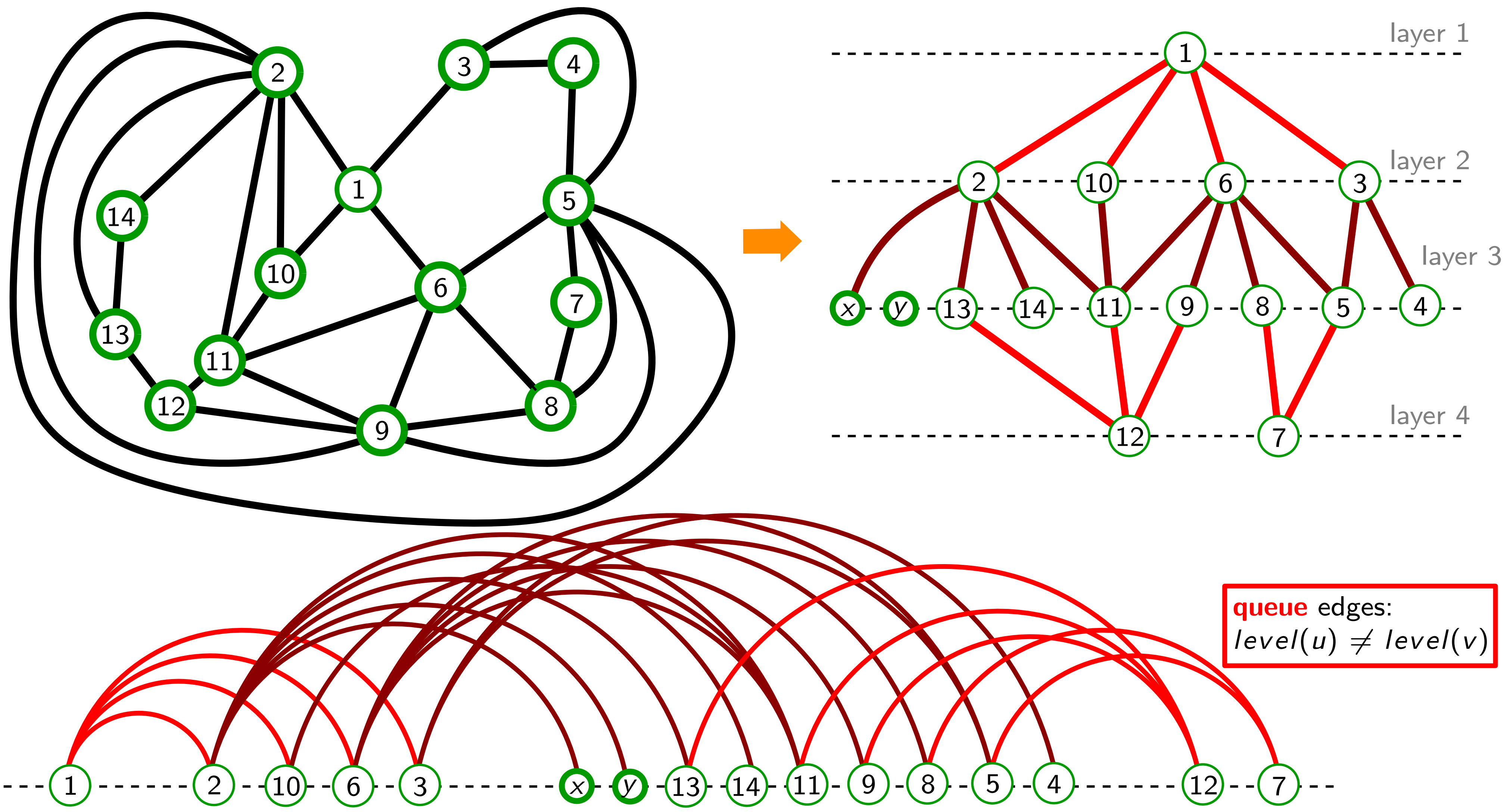

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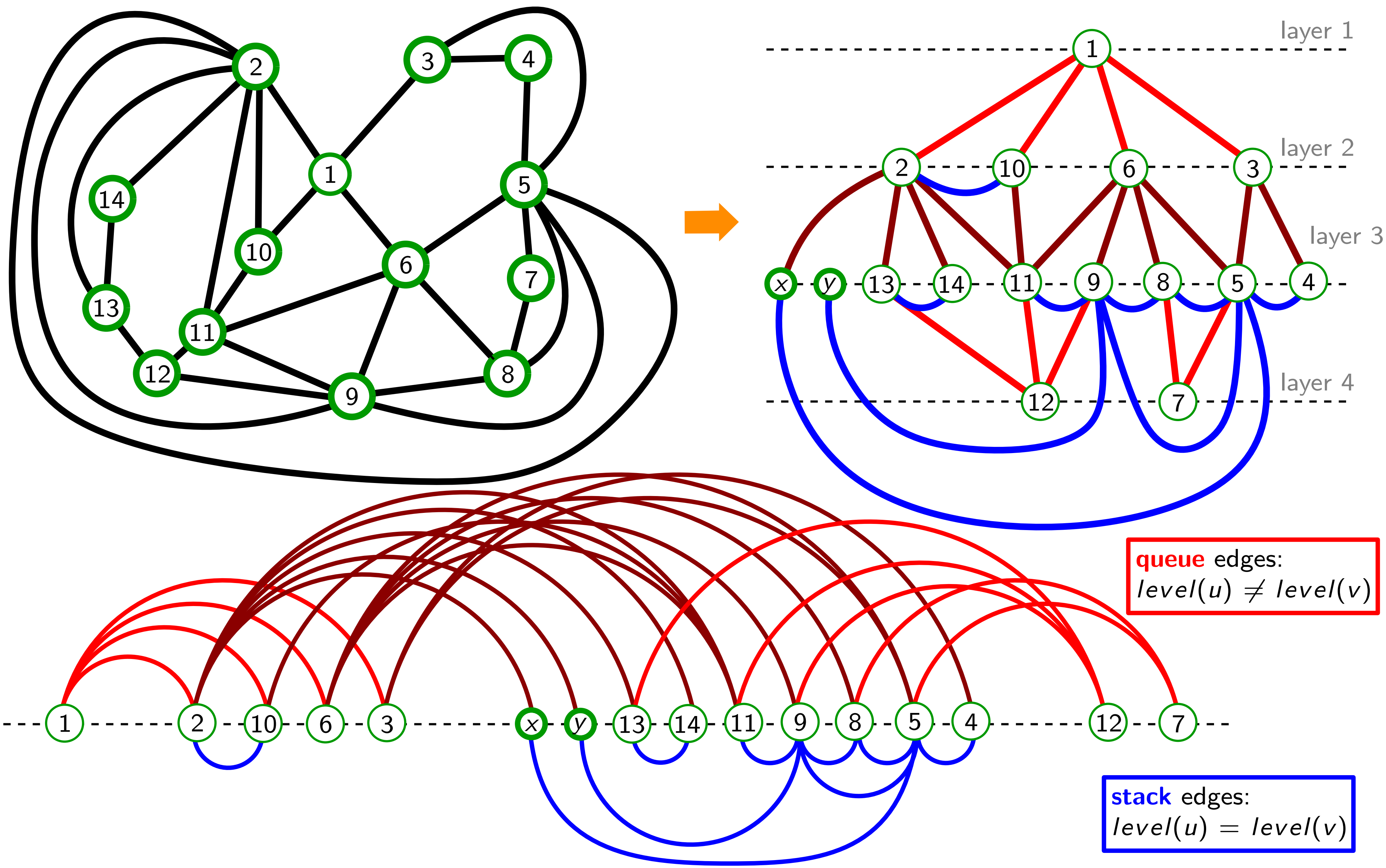

# Experiments

Goals:
- Compare 2-stack, 2-queue, and mixed 1-stack 1-queue layouts

Setup:
- Optimal solver for linear embedding (SAT+Glucose)
  `http://be.cs.arizona.edu`
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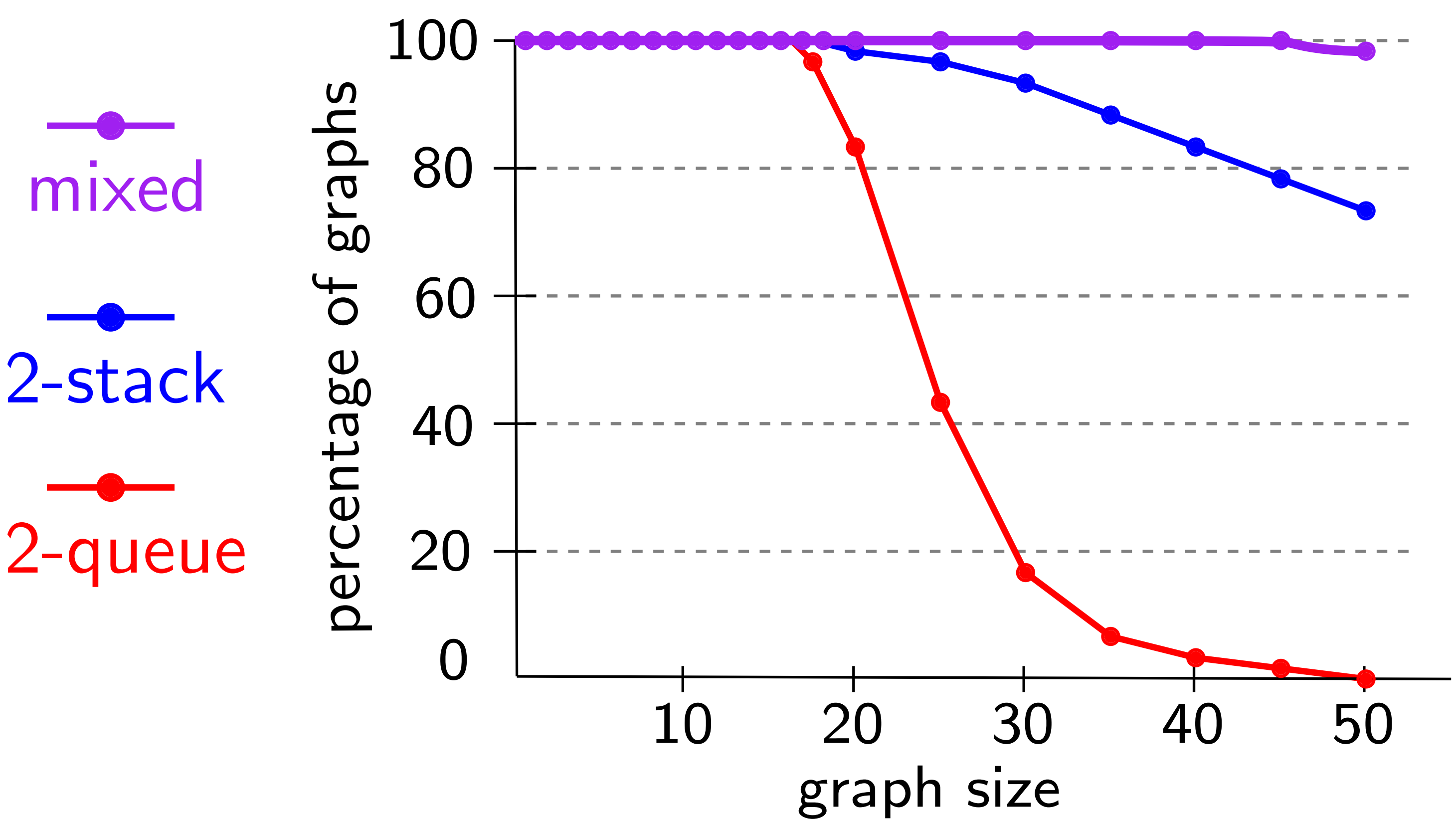

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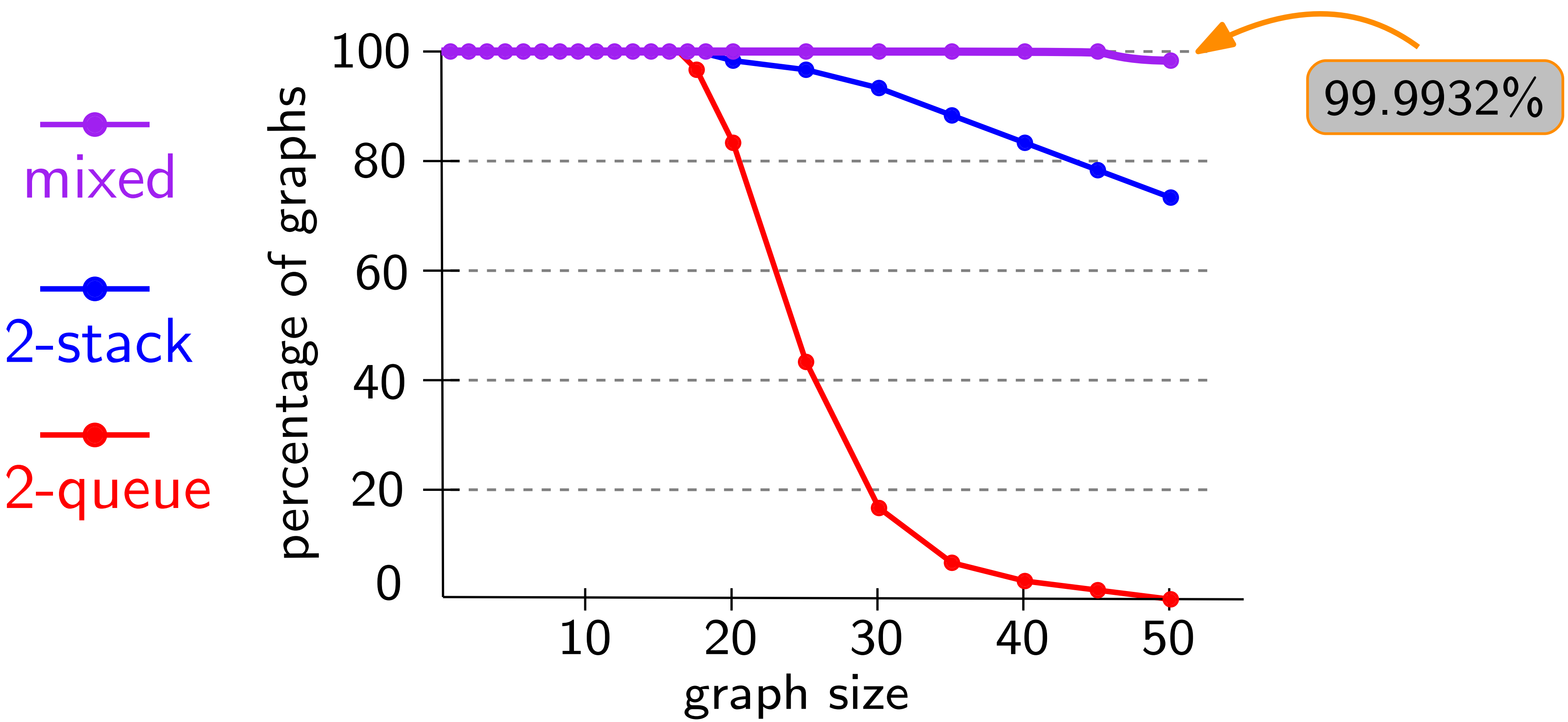

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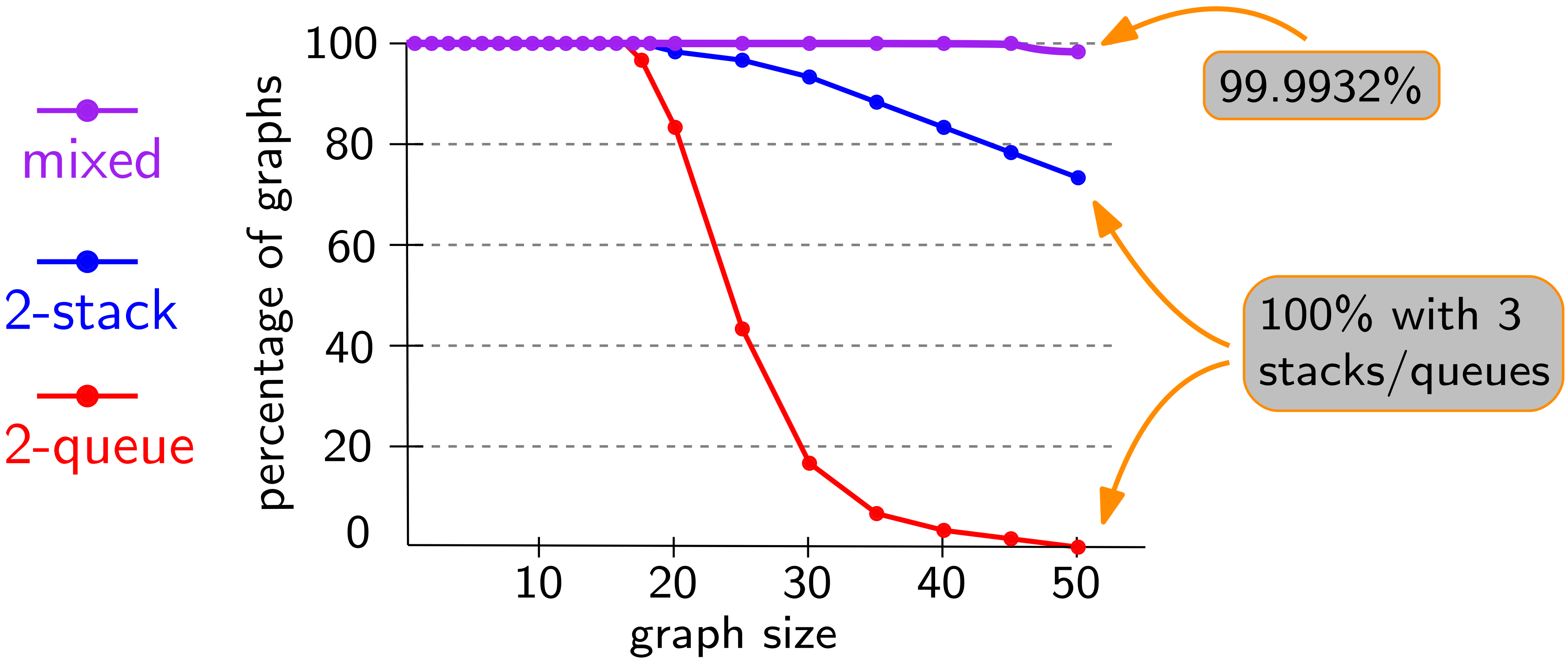

# Future Work

- Not every planar graph admits a mixed layout but every 1-subdivision does

**Open Problem 1**

What subclasses of planar graphs admit mixed layouts?

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Do 2 stacks + 1 queue or 1 stack + 2 queues suffice for all planar graphs?

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# Questions?